

**GENEVA  
GRADUATE  
INSTITUTE**

INSTITUT DE HAUTES  
ÉTUDES INTERNATIONALES  
ET DU DÉVELOPPEMENT

GRADUATE INSTITUTE  
OF INTERNATIONAL AND  
DEVELOPMENT STUDIES

# SAOMs

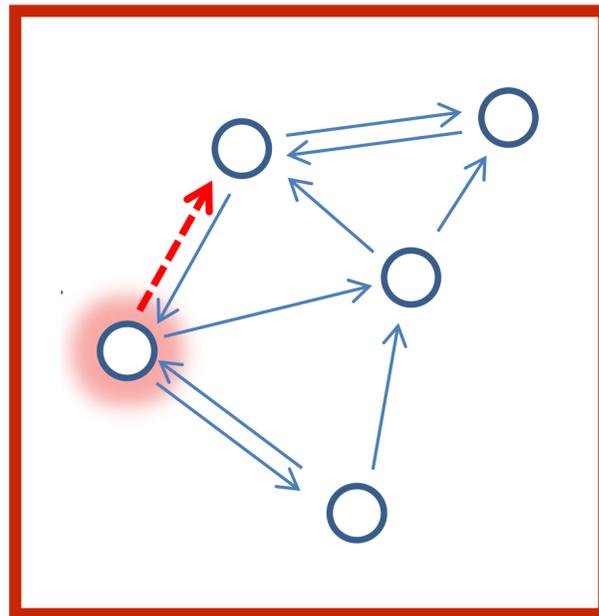
James Hollway

# Feedback on midterms

- Generally very well done
- Reminder: choice of centrality measure should be well motivated
- Reminder: transitivity for undirected networks is just closed triangles
- Reminder: which community detection algorithm produces highest modularity and/or most interpretable/sensible results
- Reminder: nodes in structural holes are called brokers; ties linking communities are called bridges
- Surprising not to see more positional and/or topological analysis...

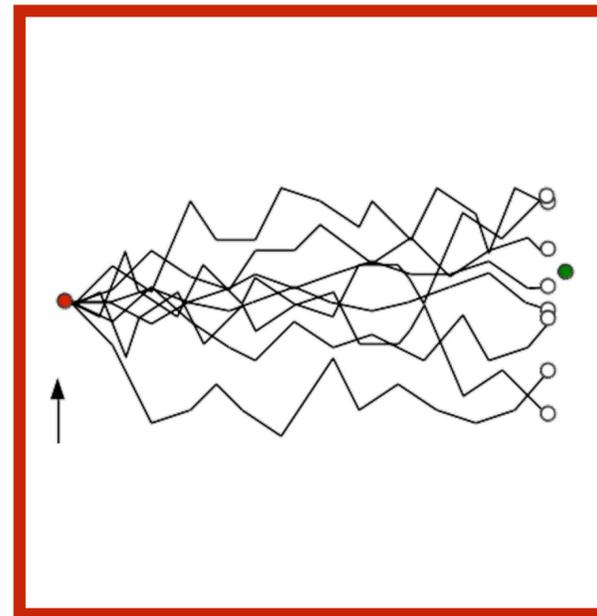
# SAOM

Model



Actor vs tie models

Estimation



MOM vs MLE

Influence

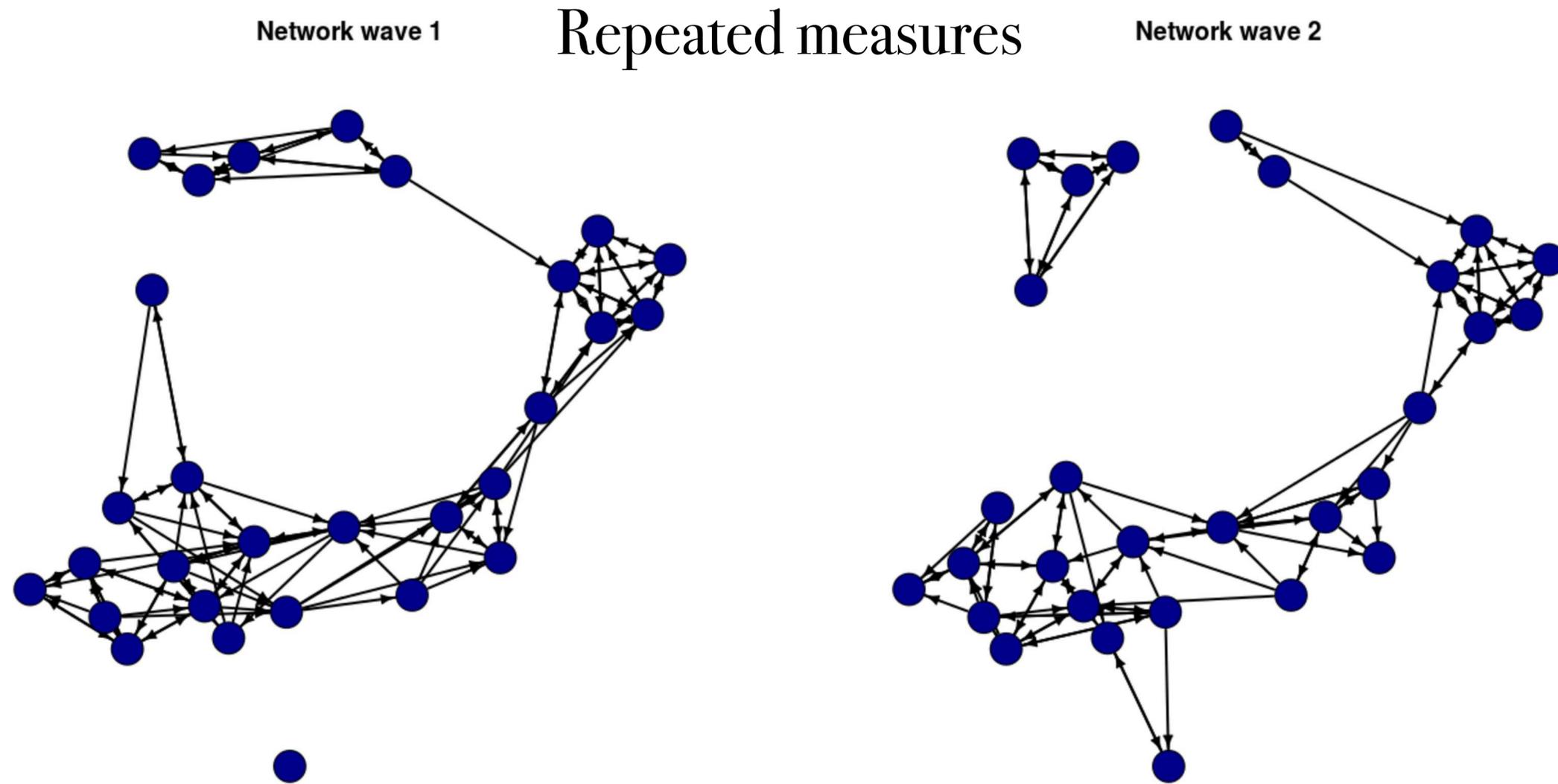


Selection vs Influence

# Why Network *Dynamics*

- Because we want to know *why* there are associations
  - Say, why are depressed people more likely to have depressed friends (Schaefer et al 2012)
- *Competing explanations* tend to involve *dynamic mechanisms*:
  - because depressed adolescents prefer depressed friends
  - because they are avoided by non-depressed people
  - because they withdraw from friendly interactions which destroys all other friendships
  - because depression is contagious along friendships

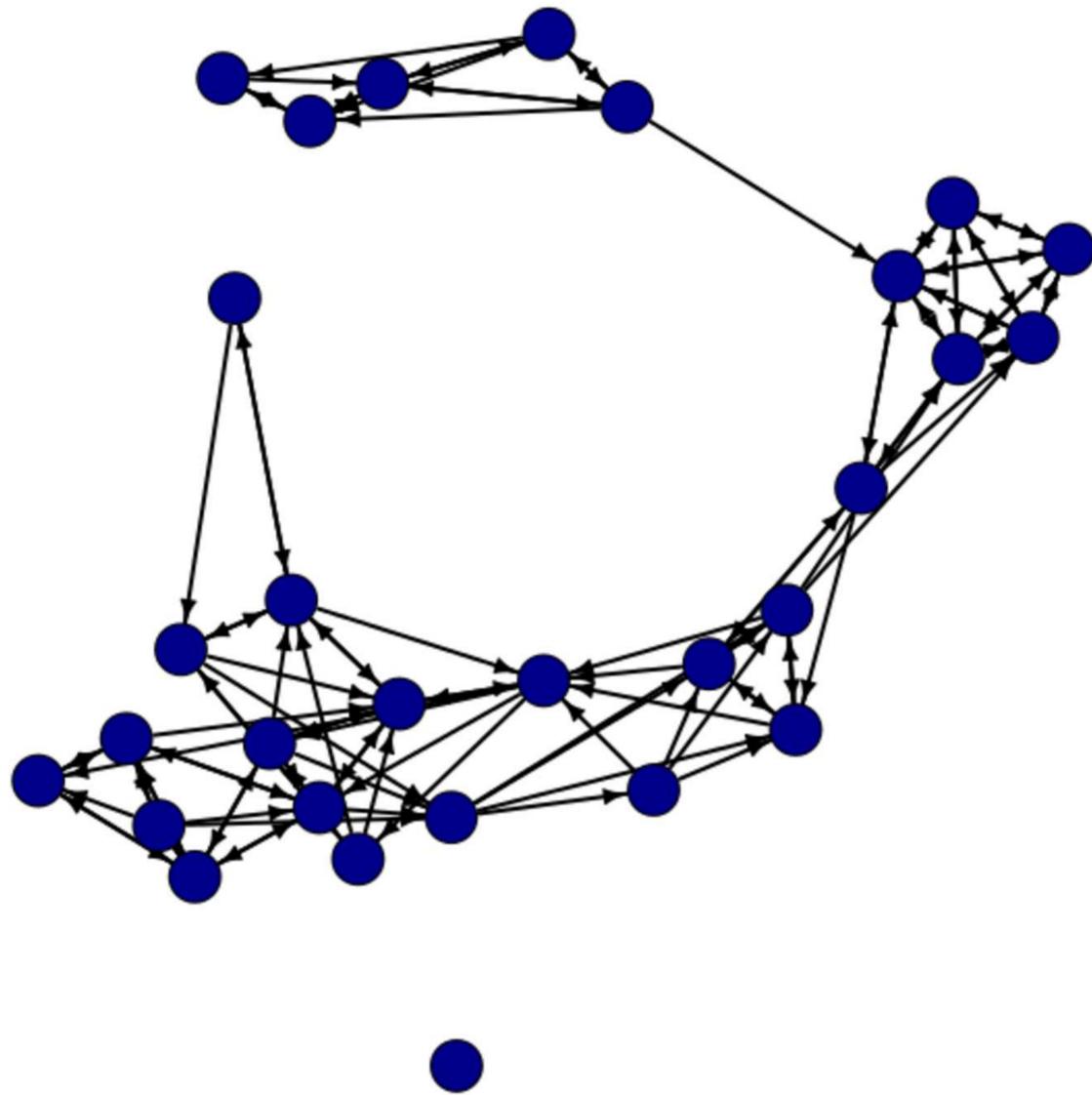
# Typical data: panel



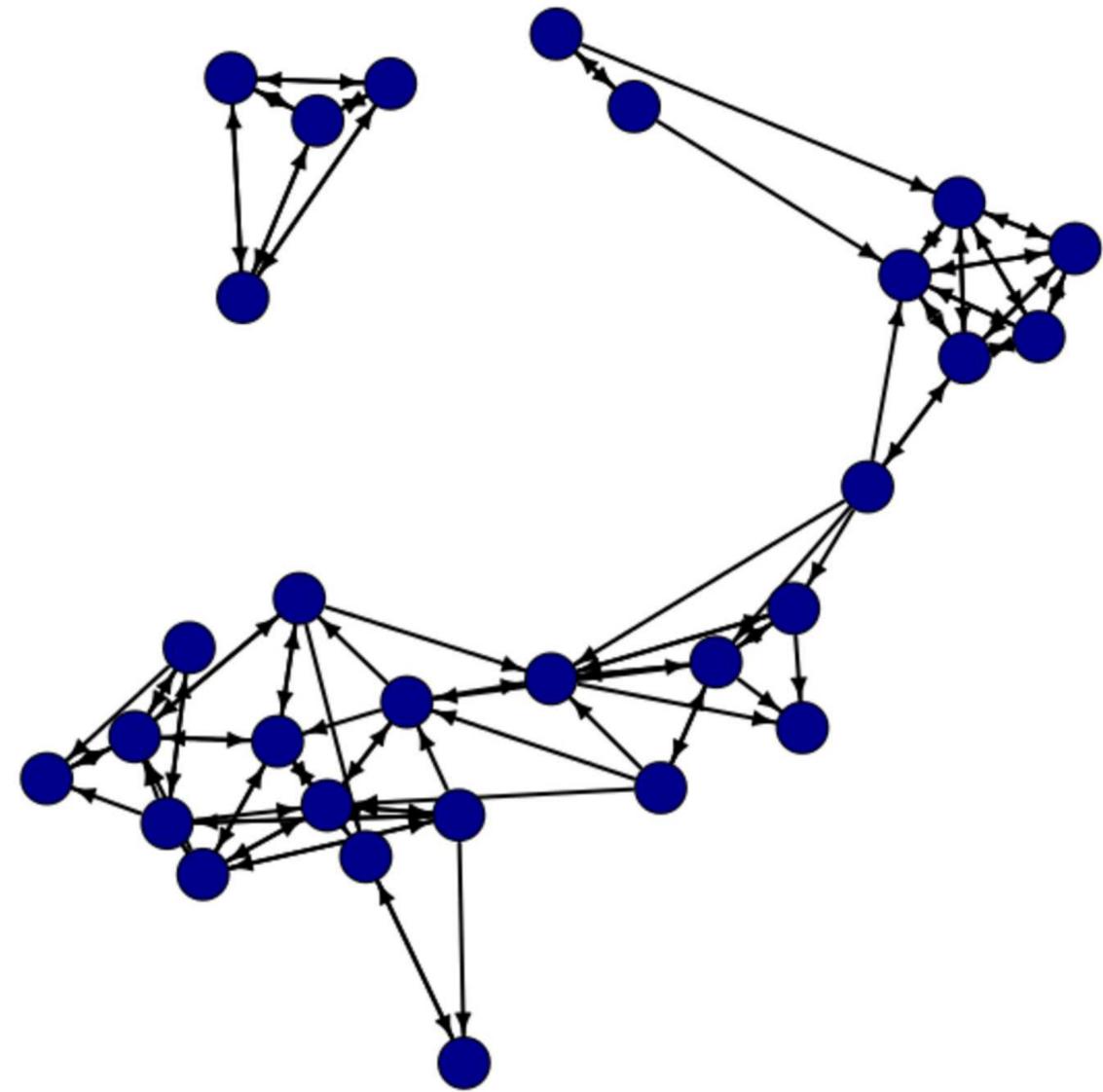
1. Same group of actors (some **composition change** allowed)
2. Same relational variable (**states** not **events**)
3. Some, but not too much change

# Which forces shape this social network's evolution?

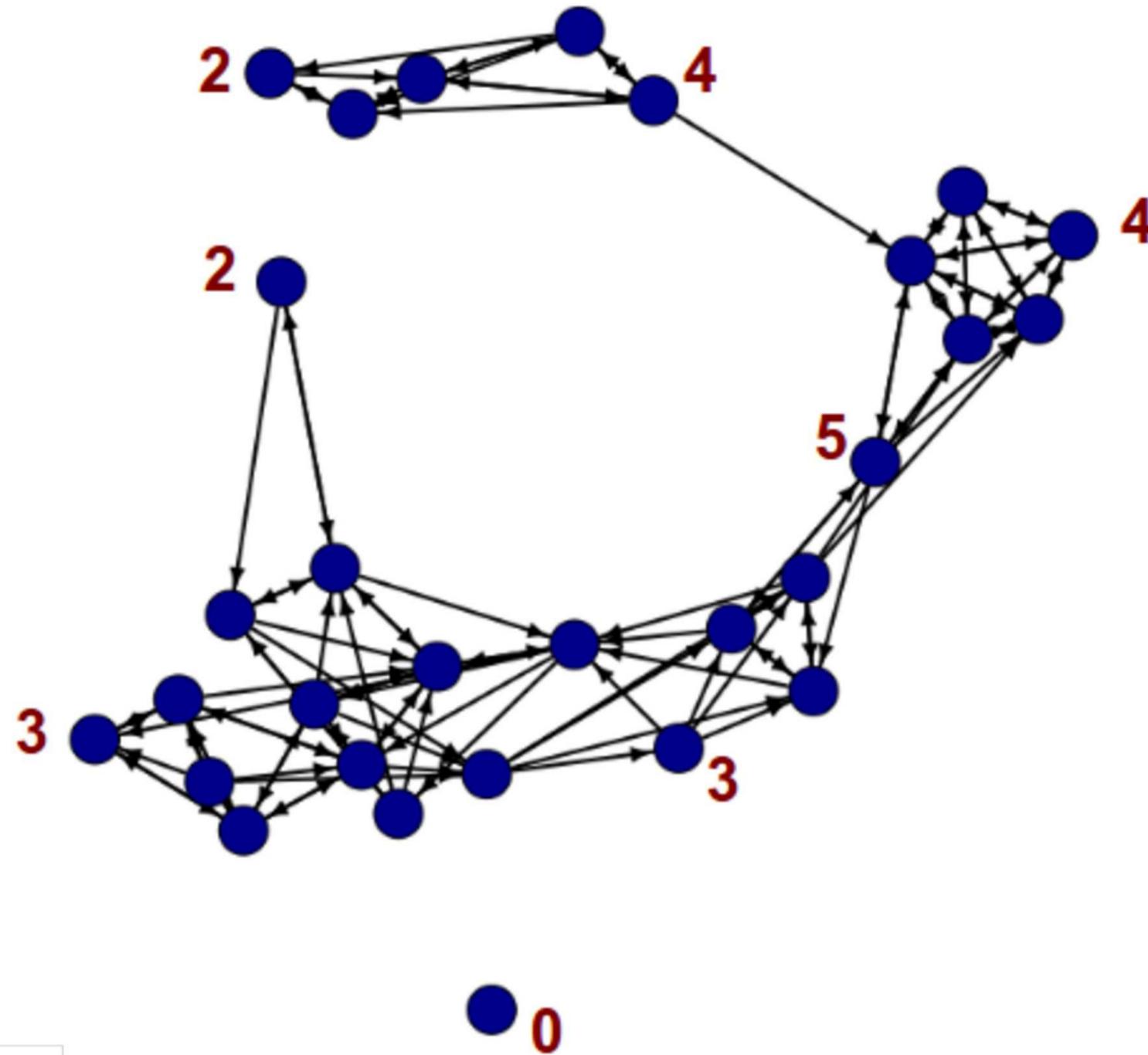
Network wave 1



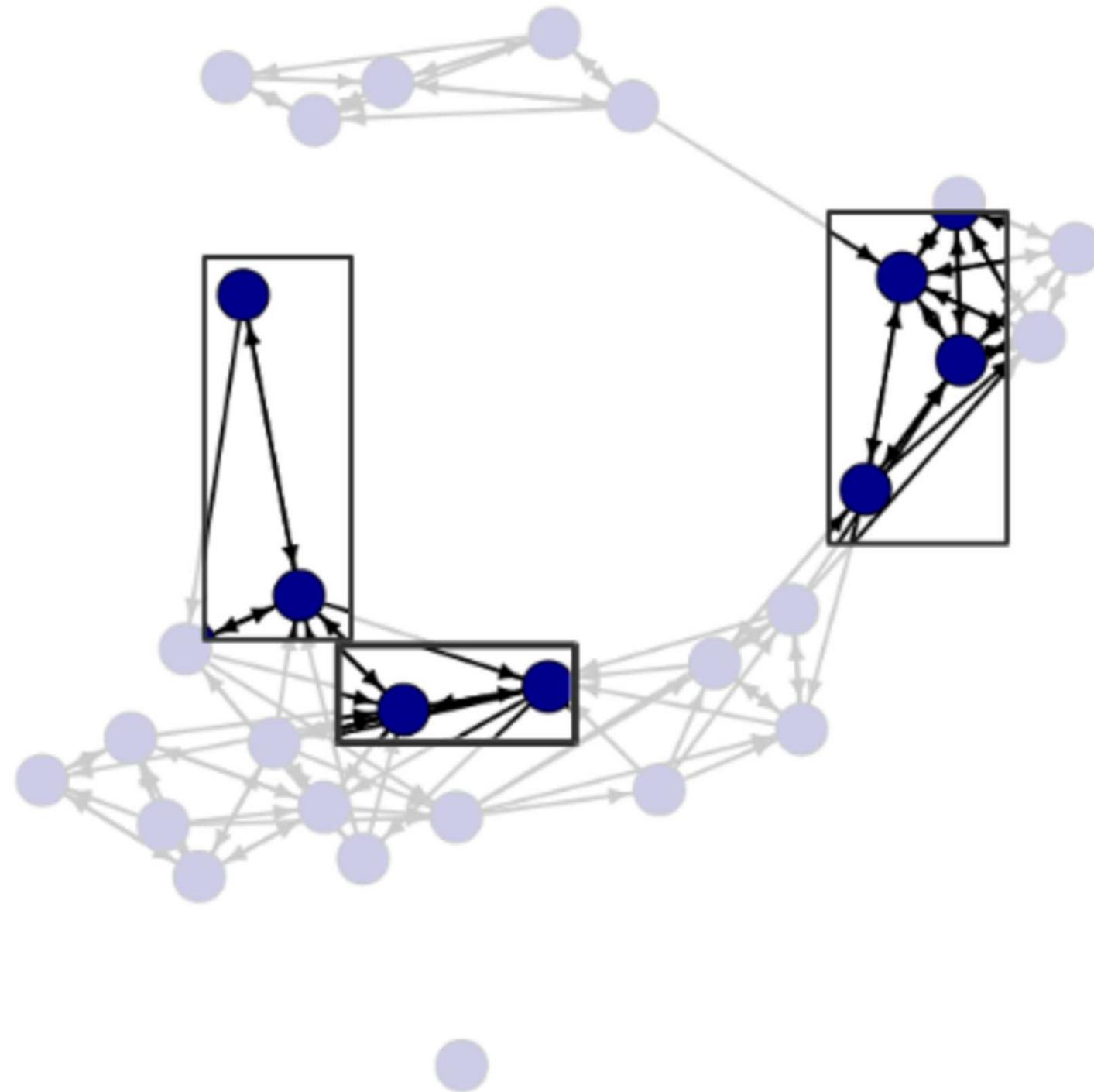
Network wave 2



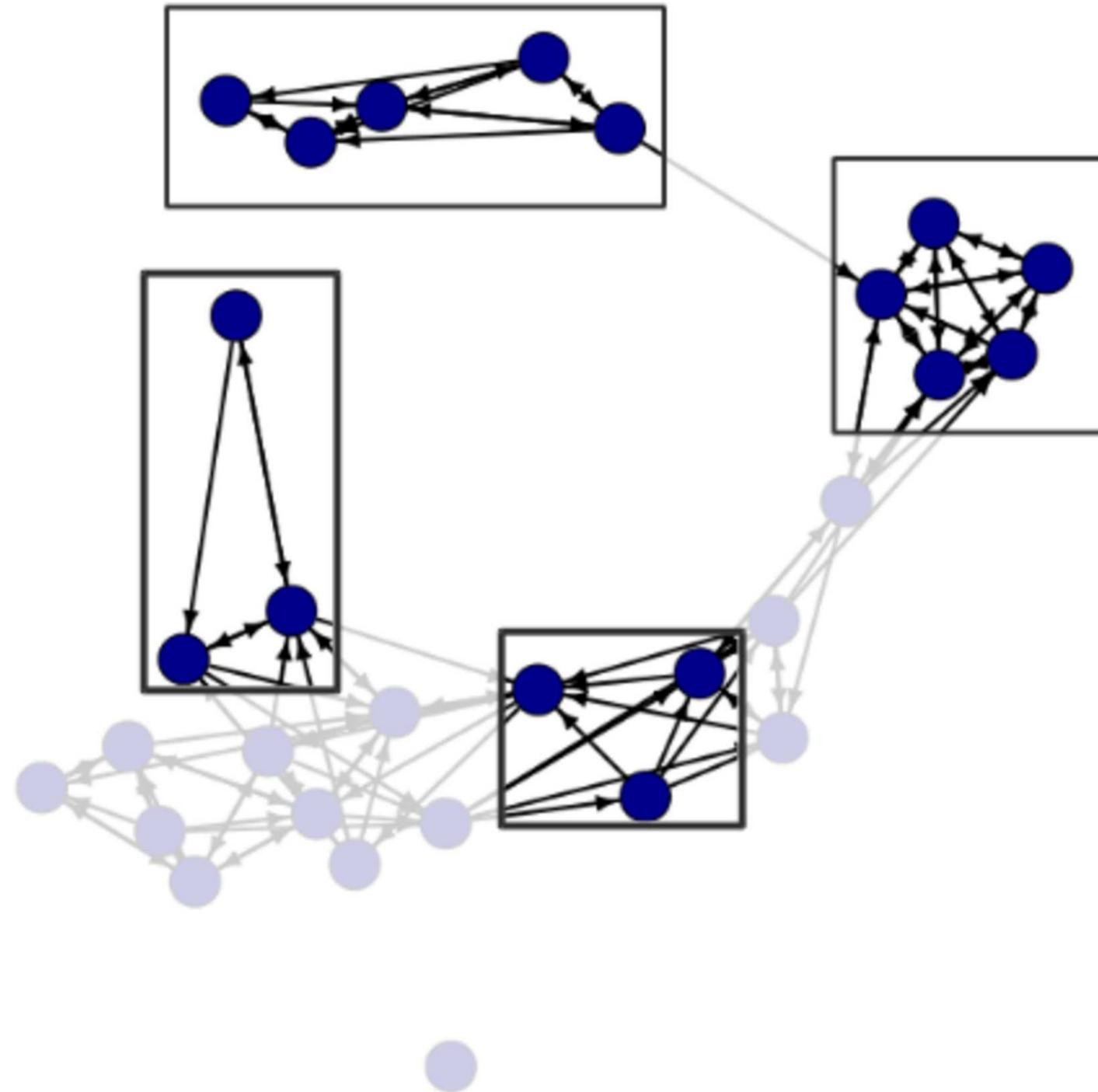
# Social network ties are costly



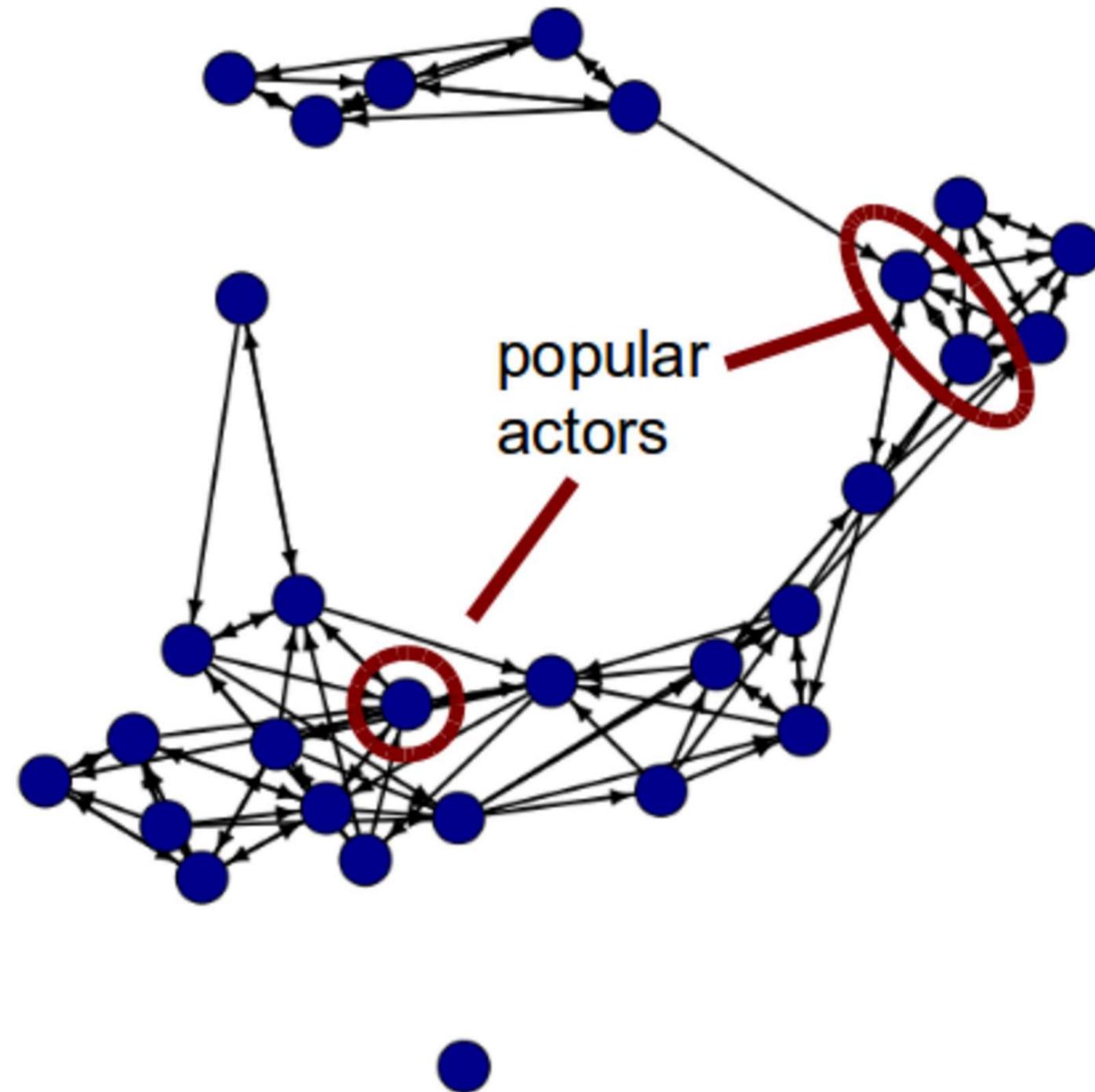
# Individuals form and maintain reciprocal ties



# Transitivity leads to clustering

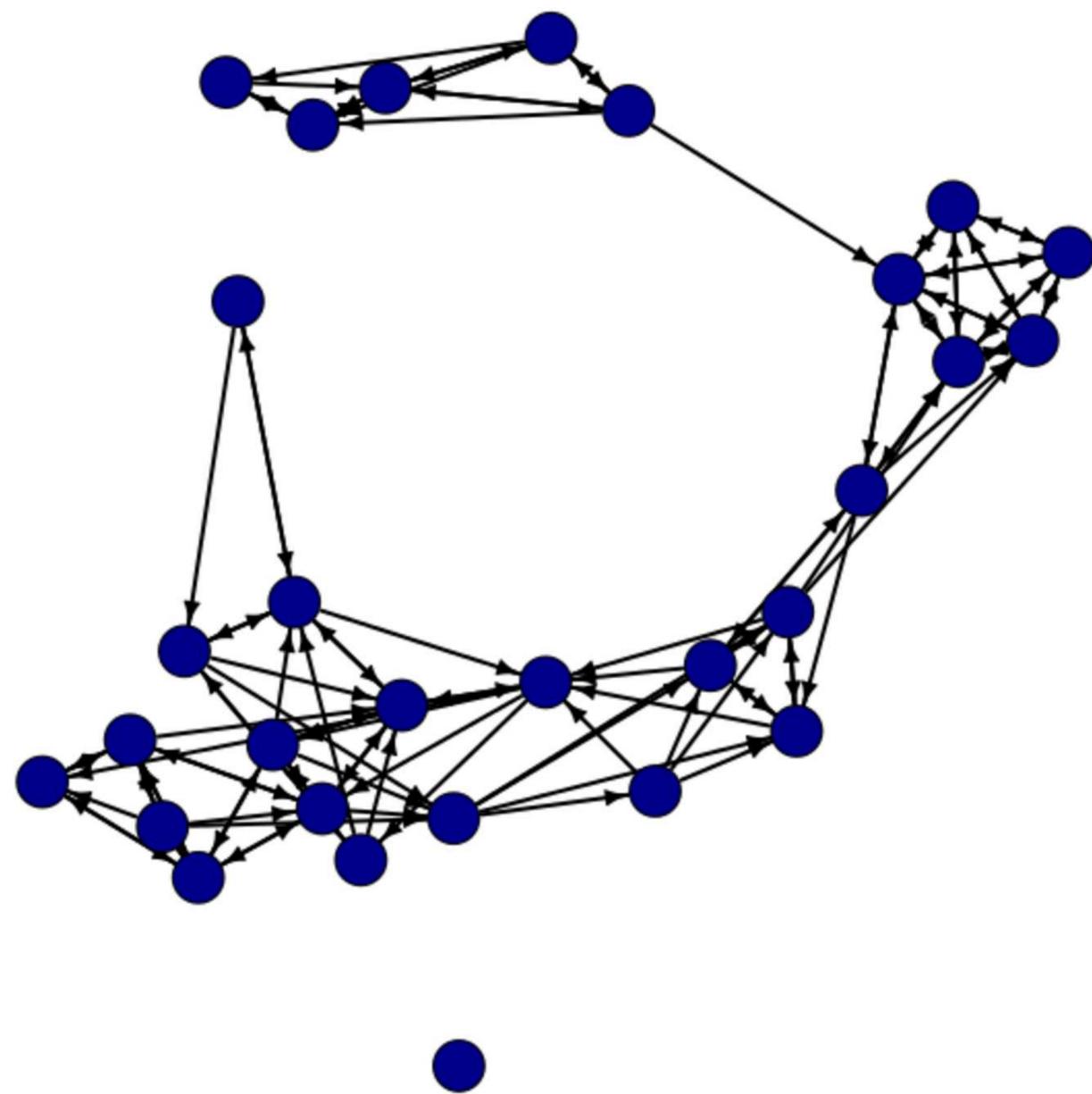


# Status hierarchy shapes friendship networks

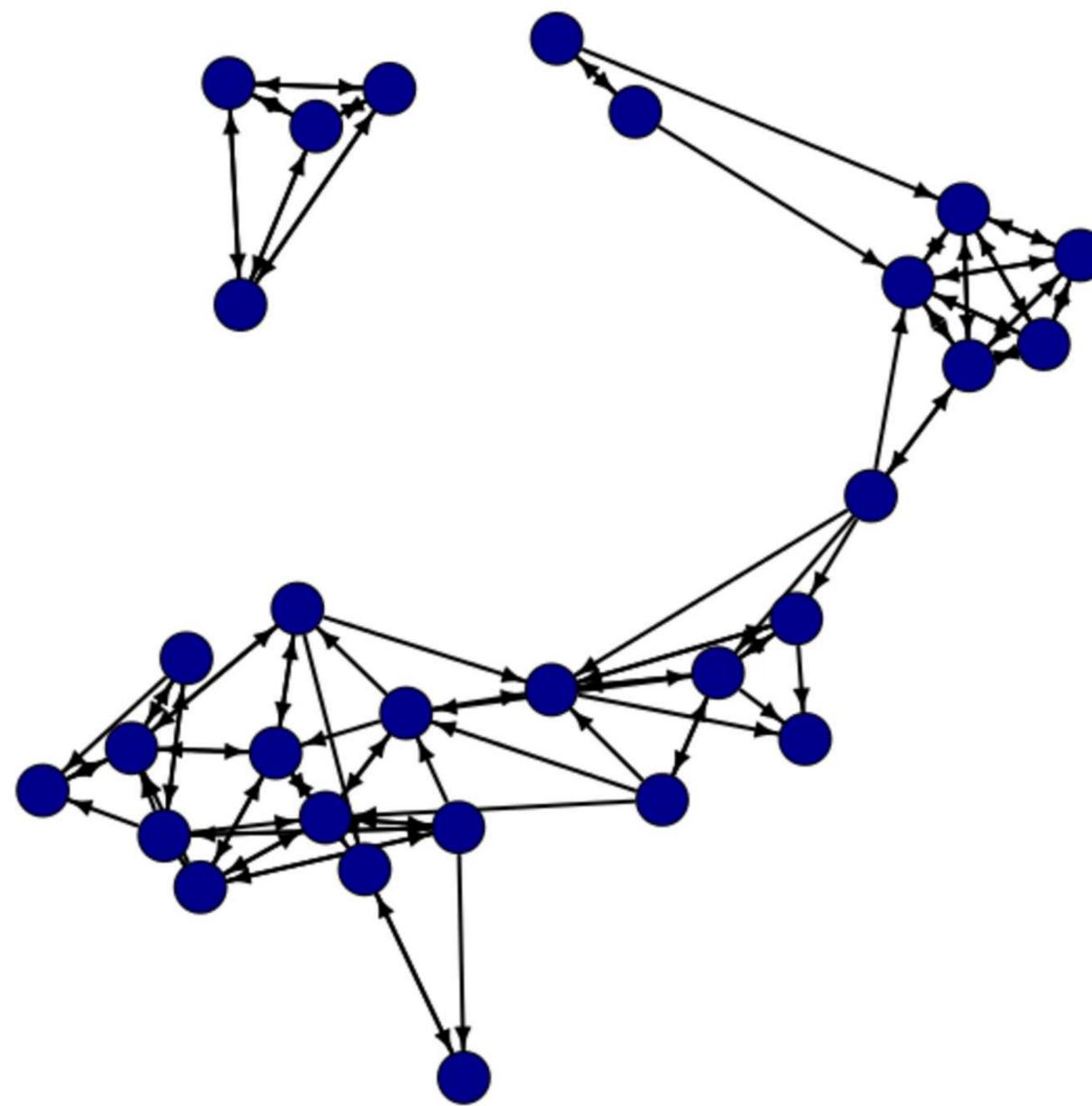


# What else?

Network wave 1

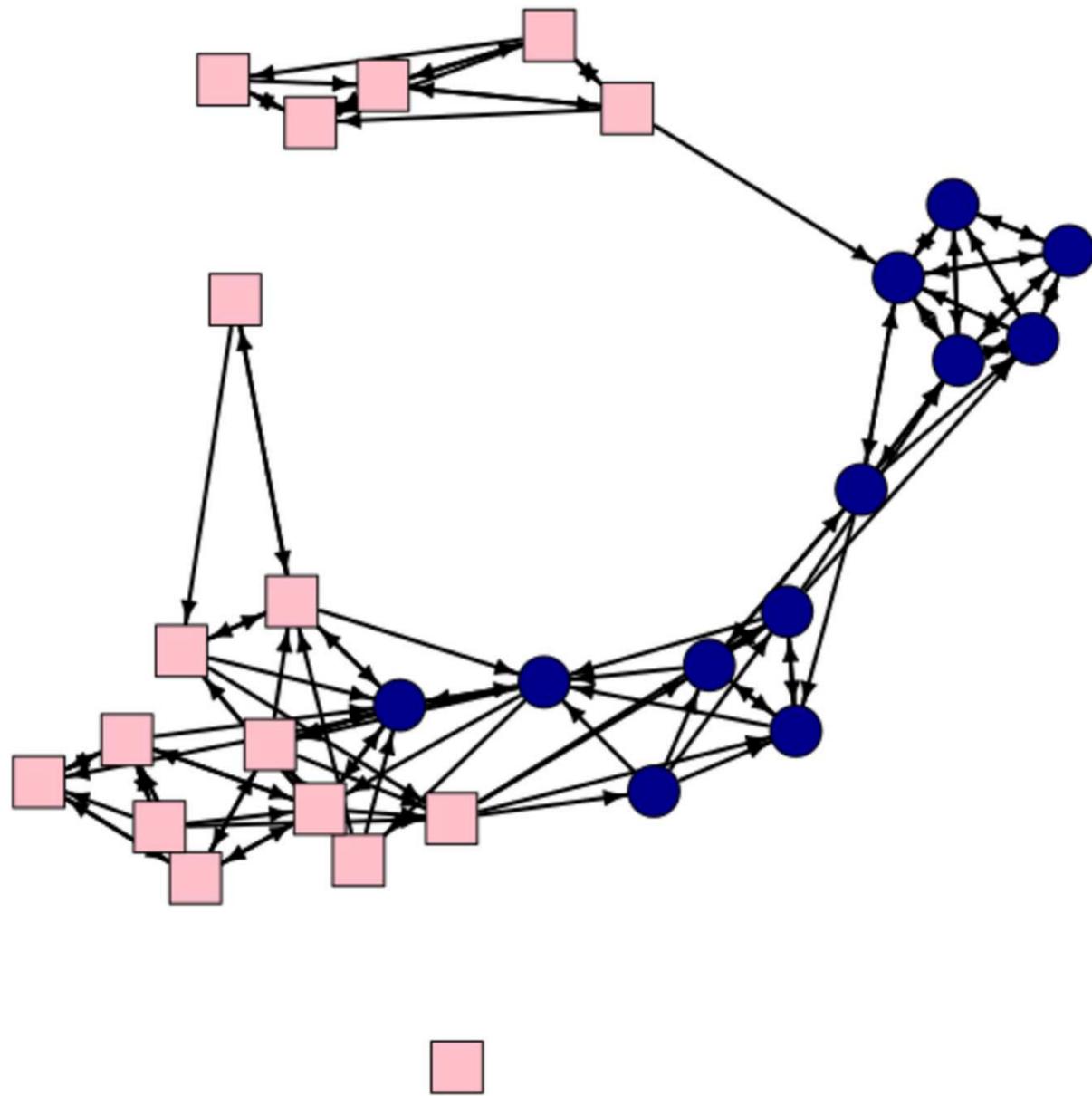


Network wave 2

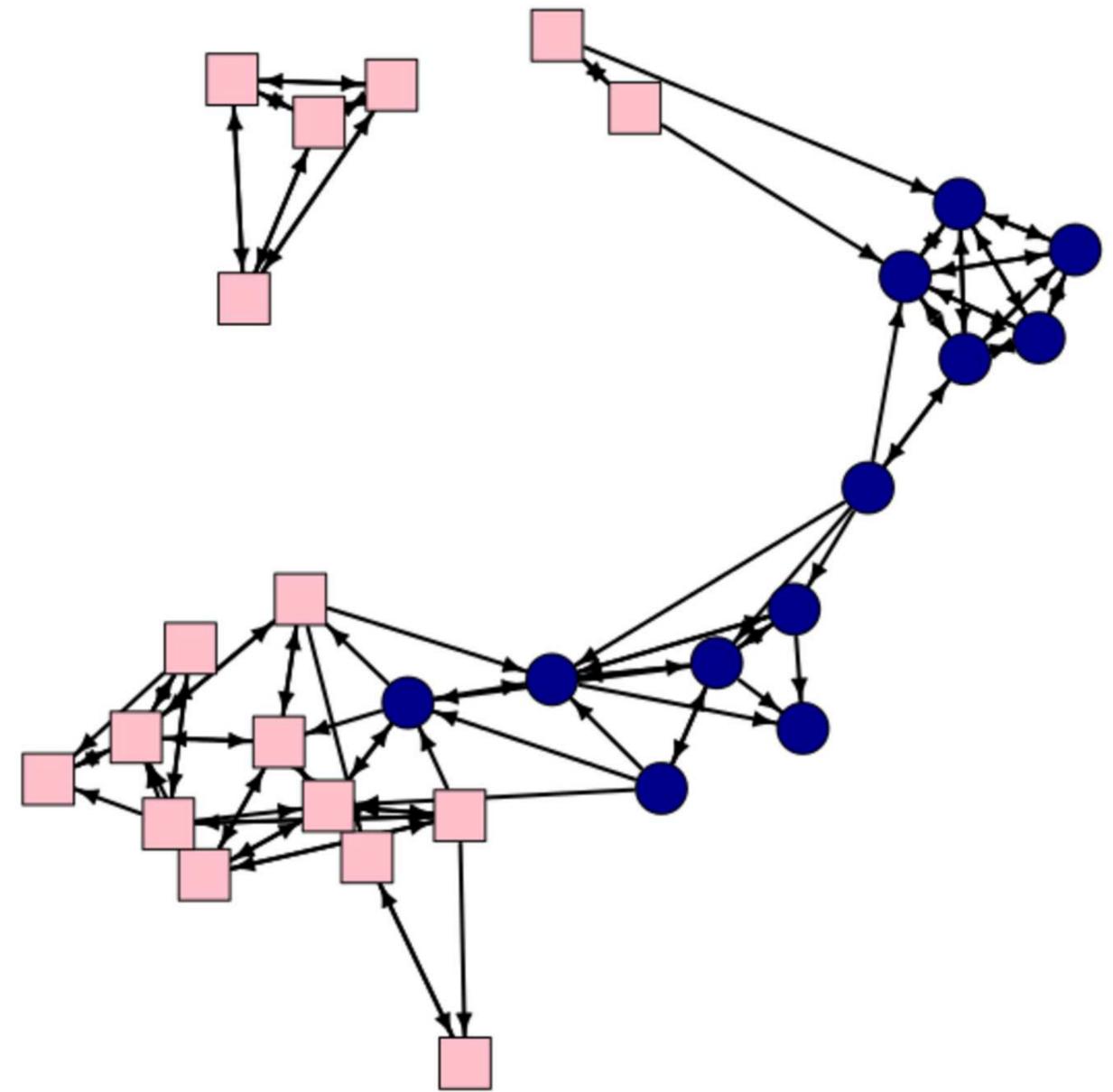


# Gender homophily?

Network wave 1

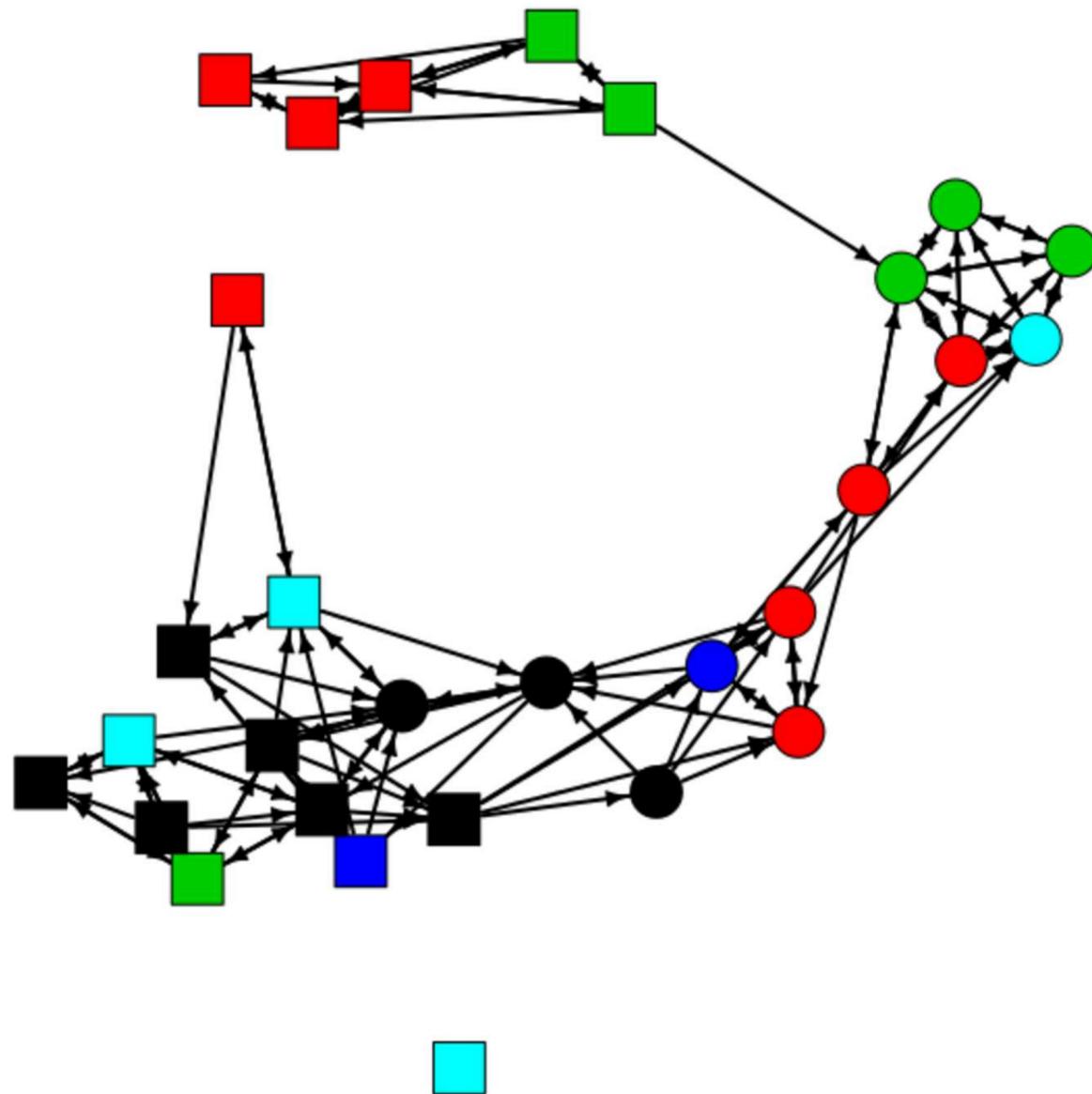


Network wave 2

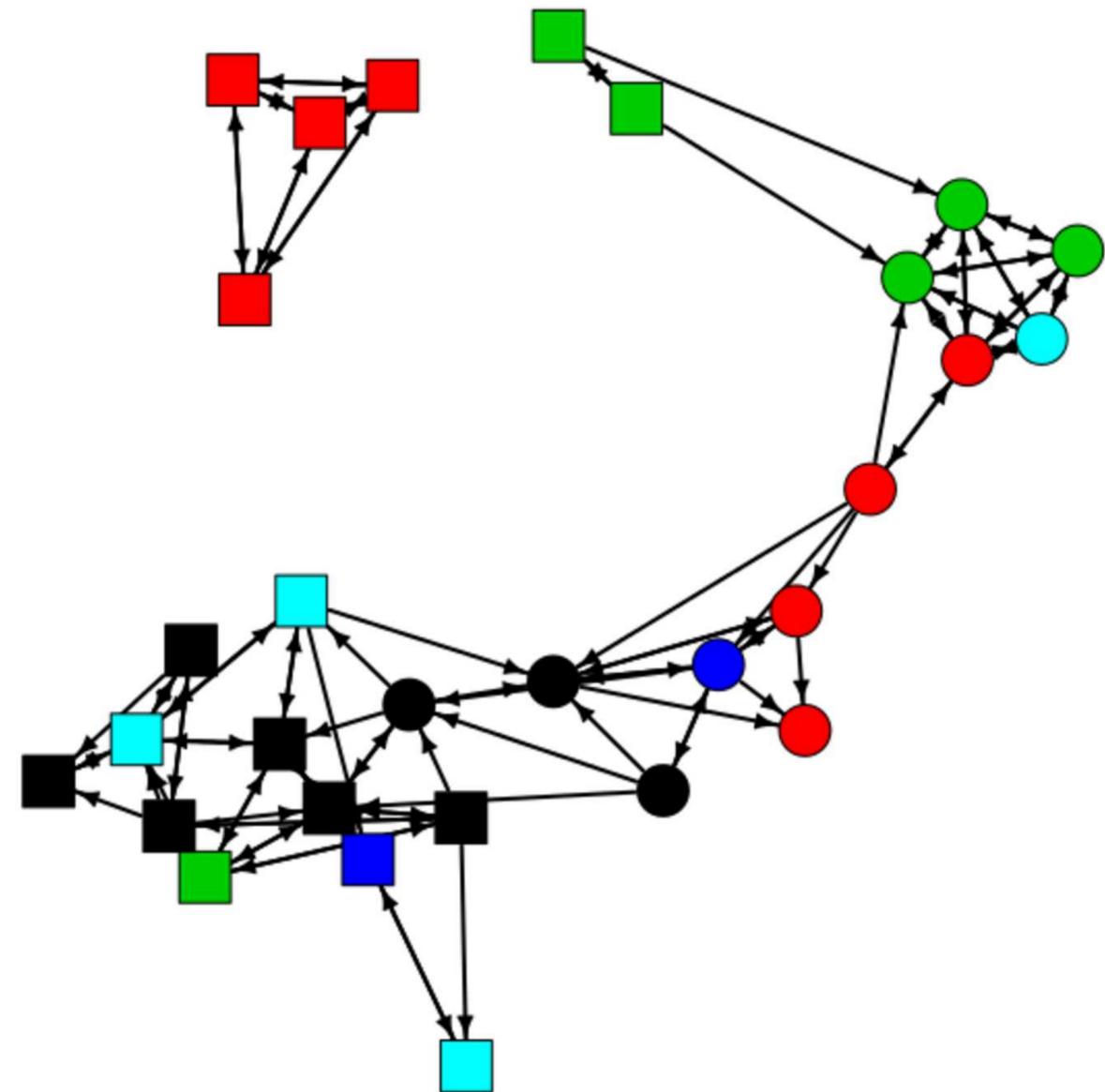


# Ethnic homophily?

Network wave 1



Network wave 2

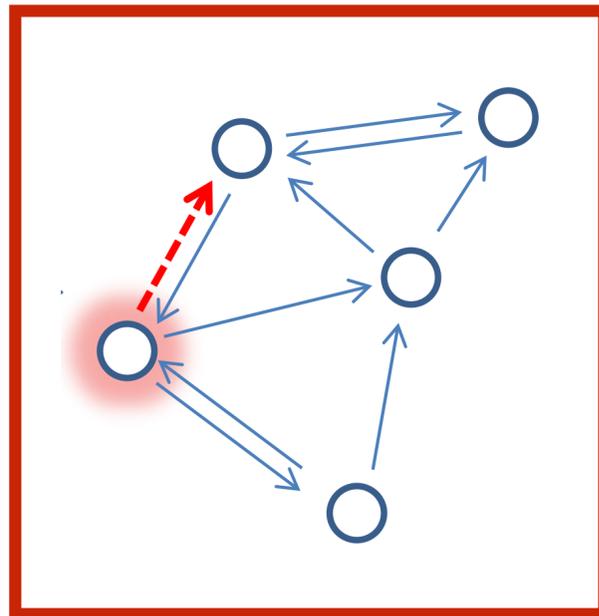


# Modelling thoughts

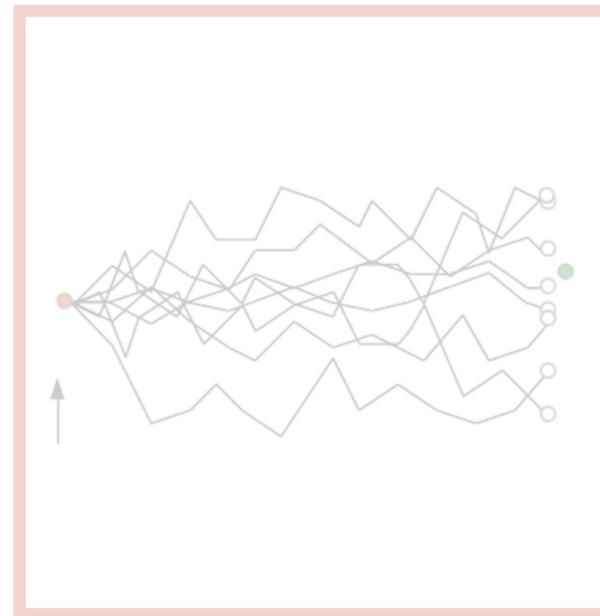
- A **statistical approach** is necessary to control for alternative explanations
- A **complete network approach** is necessary because selection can only be studied when the complete pool of candidates is known
- A **longitudinal approach** is necessary to link antecedents with consequences
- A (weak: see Udehn 2002) **methodologically individualist approach** is useful for bringing the model close to theory

# SAOM

Model



Estimation



Influence

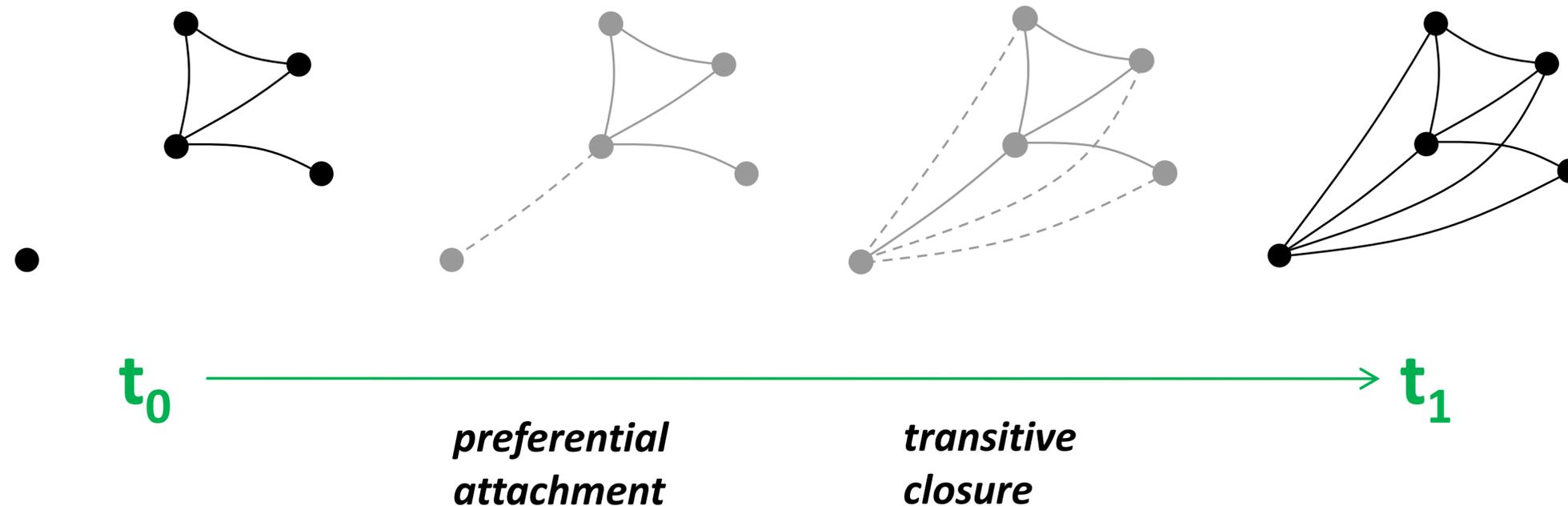


# SAOMs are not ERGMs

- SAOMs are a **continuous-time** network model
  - They model change in social networks in continuous-time using empirical panel data with SIENA (Simulation Investigation for Empirical Network Analysis)
  - See Block et al 2018
- SAOMs are an **actor-oriented** network model
  - They model change as a function of individuals' choices about whom they want to relate to and how they want to behave
  - See Block et al 2019

# Why Continuous-Time?

- Because complex patterns emerge from simple(r) mechanisms

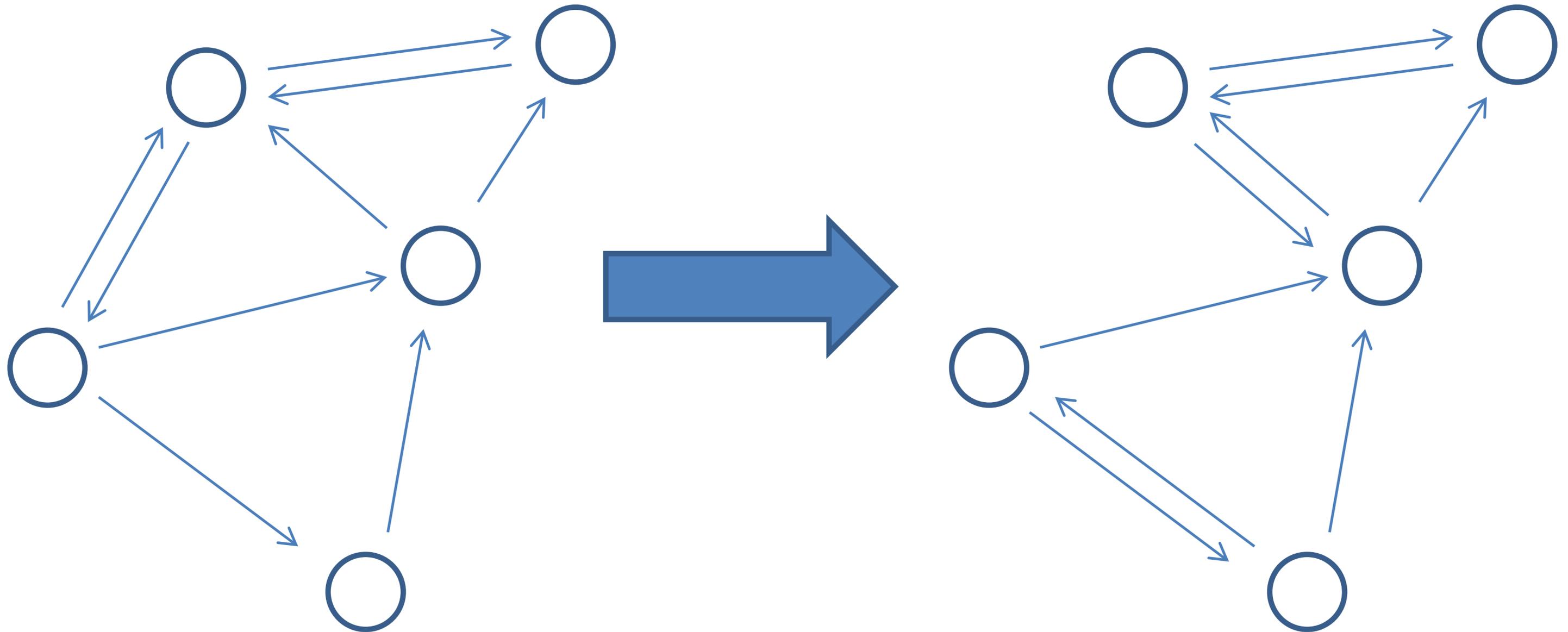


- New ties may be realisation-contingent on other new ties.
- Cannot easily model compound emergence in **discrete-time**.

# Why Actor-Oriented?

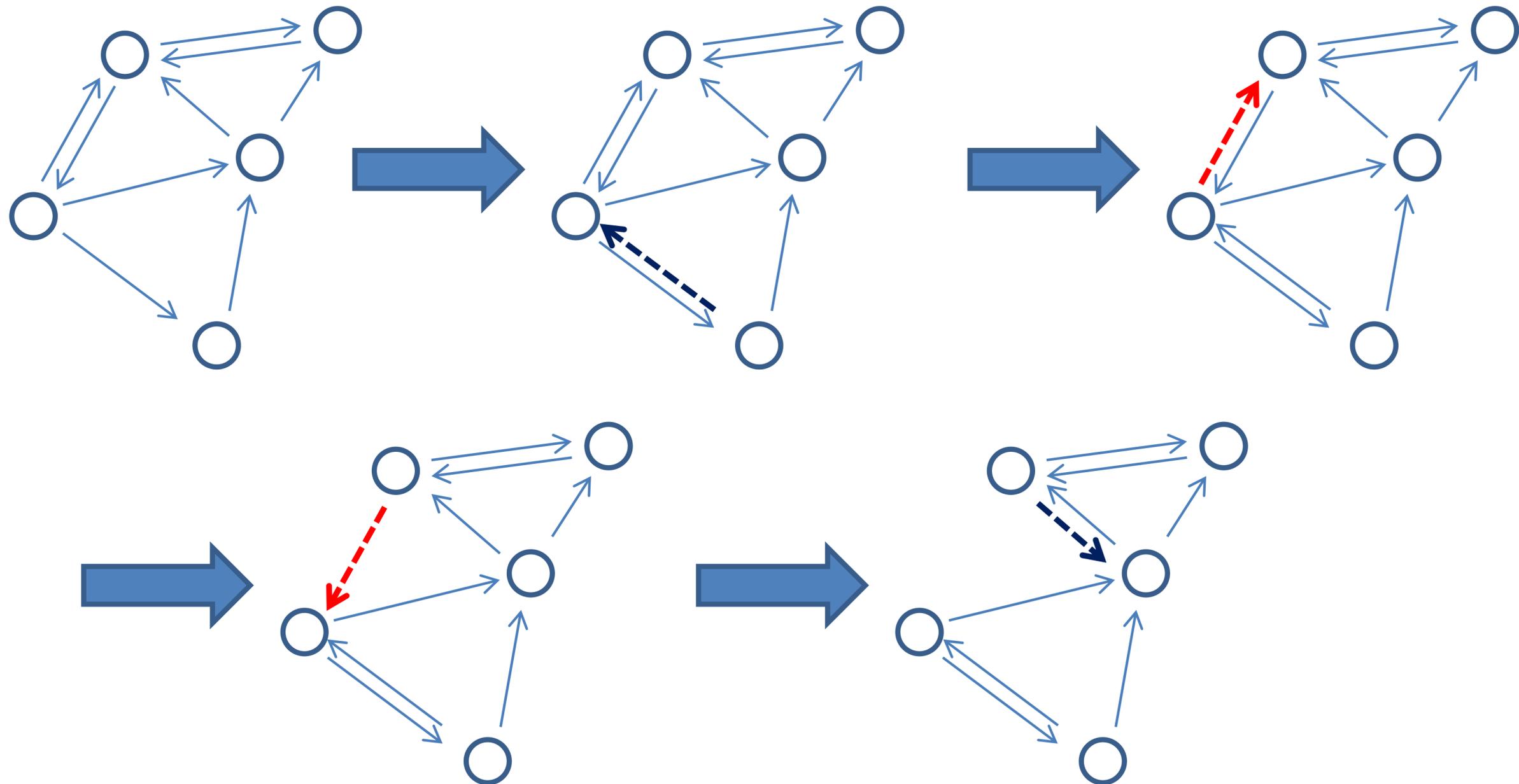
- All social network change is brought about by individual or collective **agents** that decide to send or drop a tie (homophily, withdrawal, avoidance, etc)
- As the actor is the **locus of control**, we should model the tie changes from its perspective

# Intuition

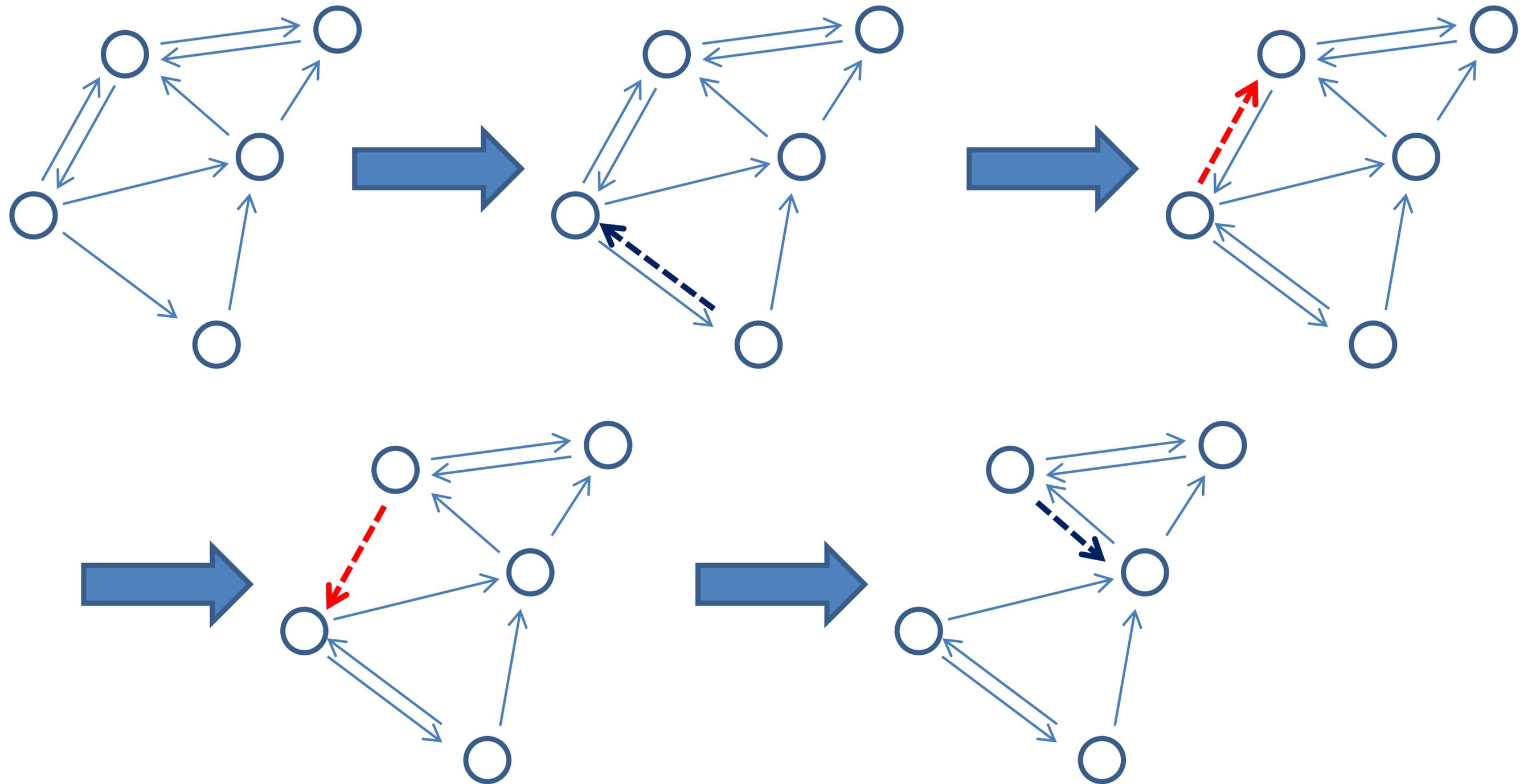


# Continuous-Time

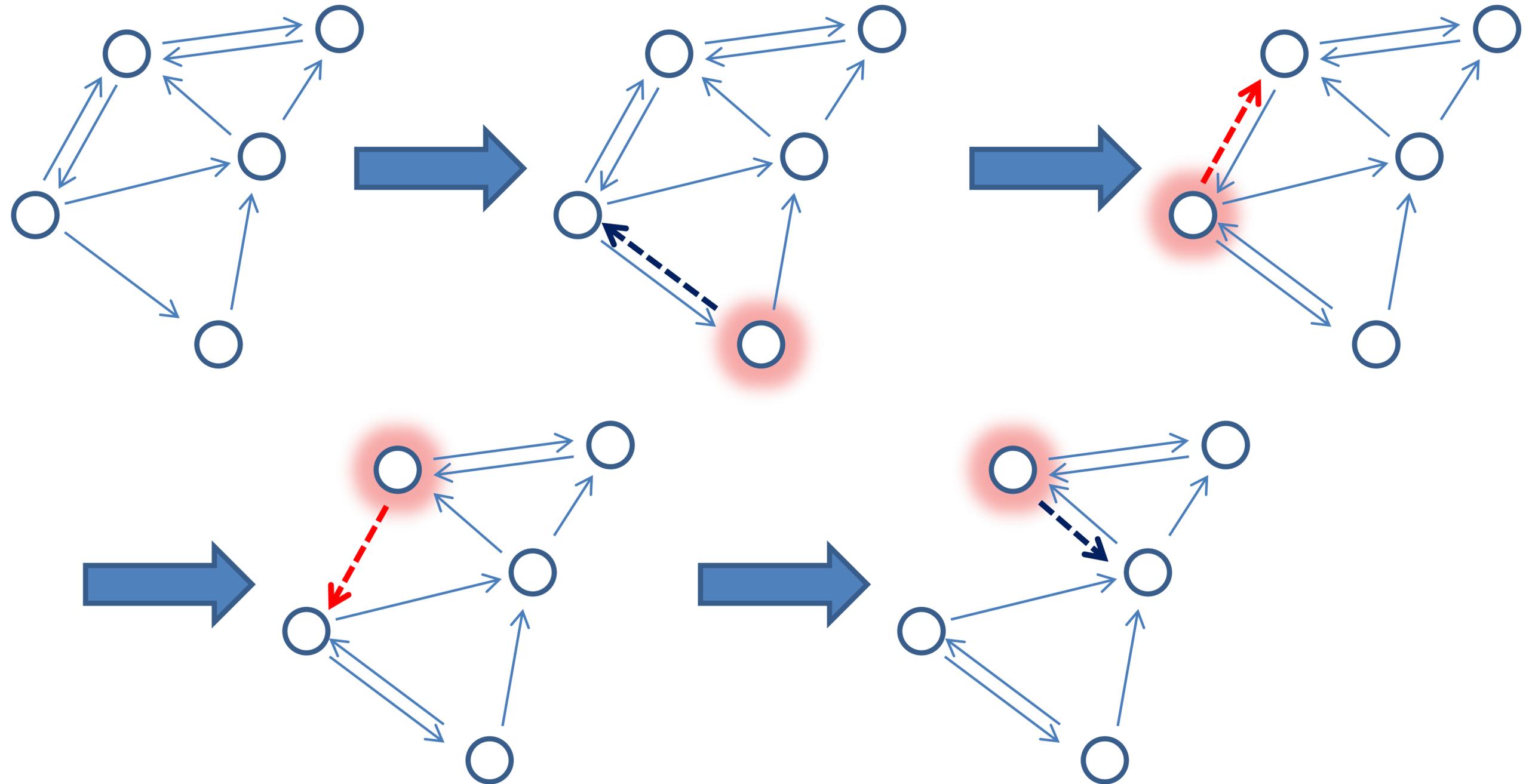
*This is one potential path how the network develops from  $t_1$  to  $t_2$*



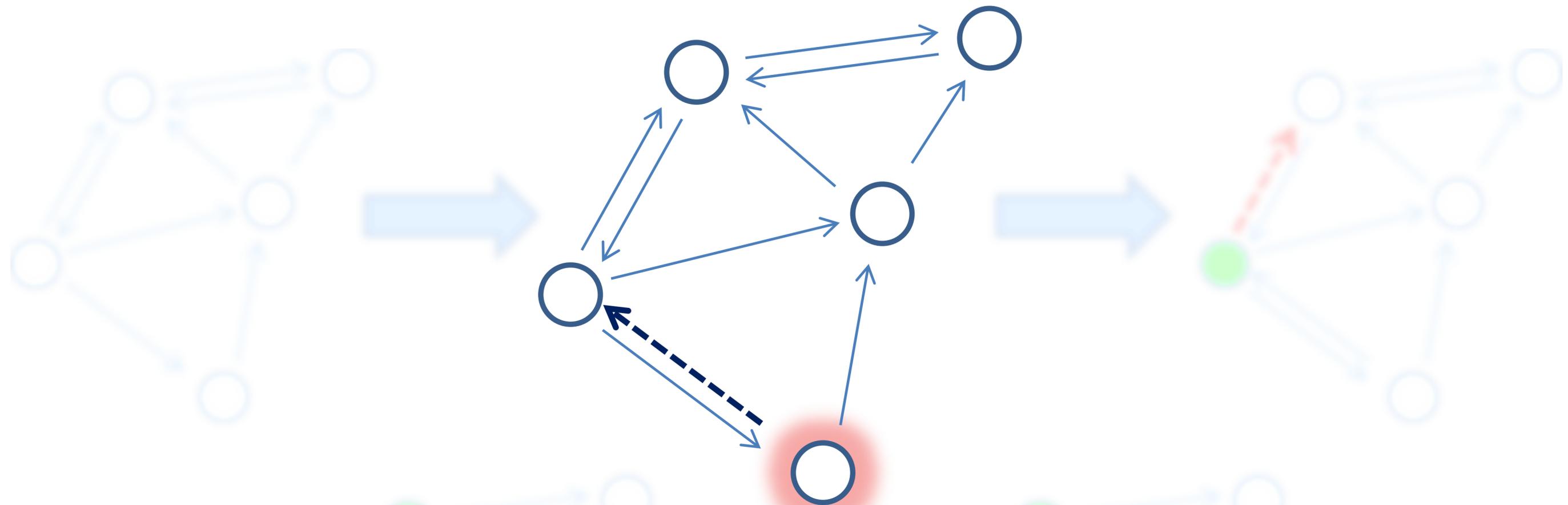
# Mini-Step



# Actor-Oriented



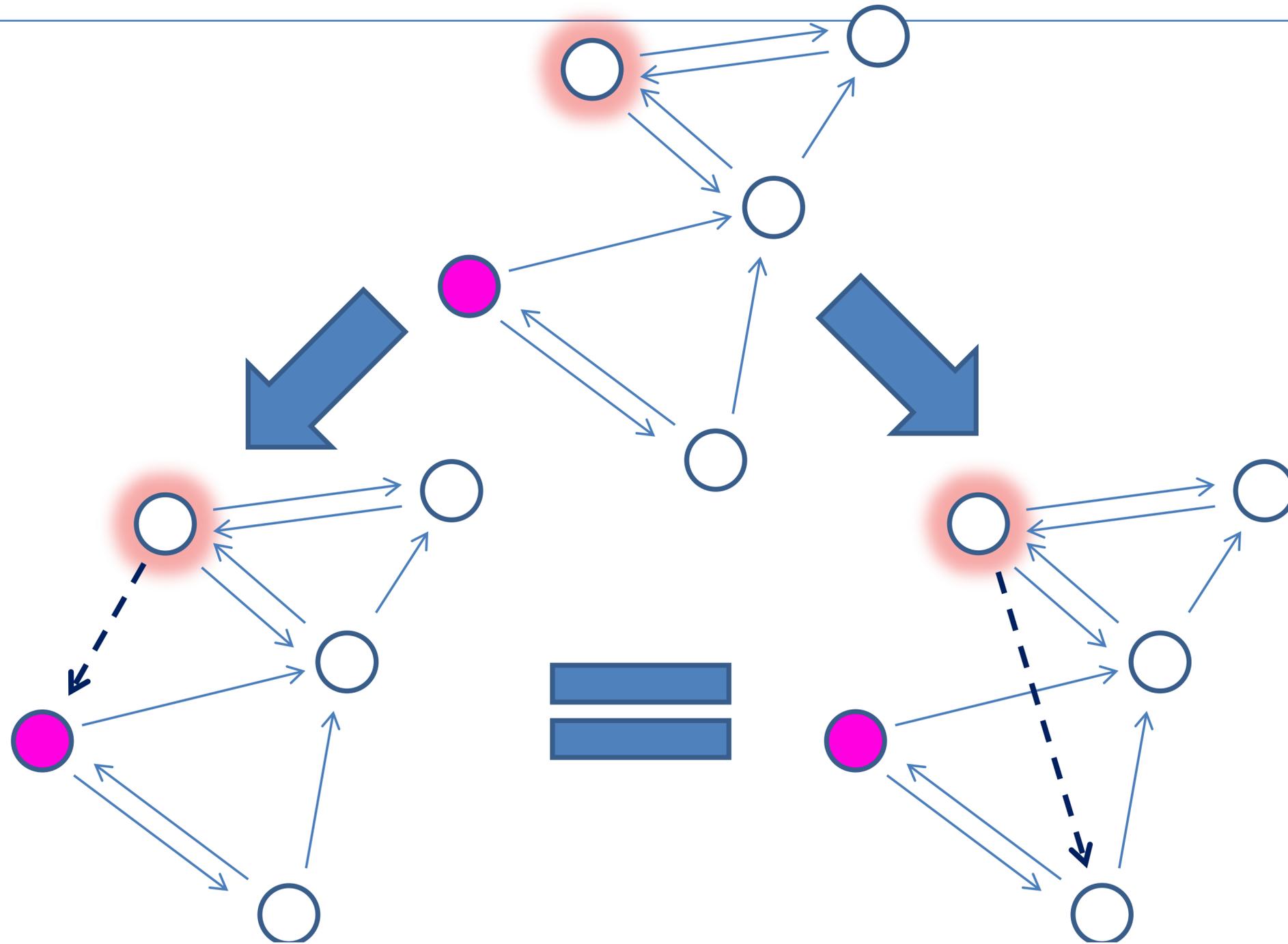
# Actor-Oriented



**The glowing actor DECIDES what tie change is most appealing.**

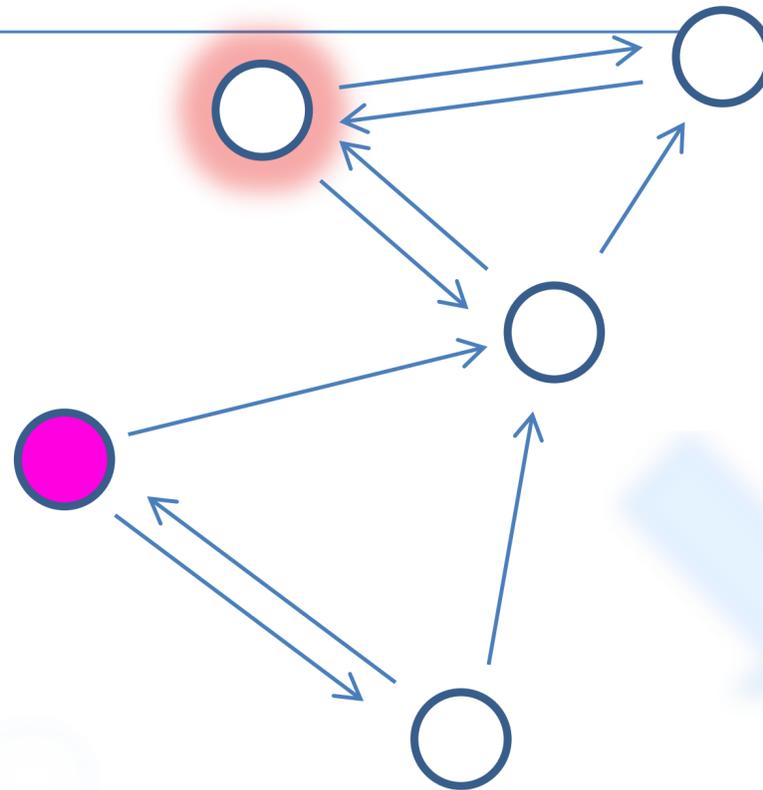
# Markov Assumption

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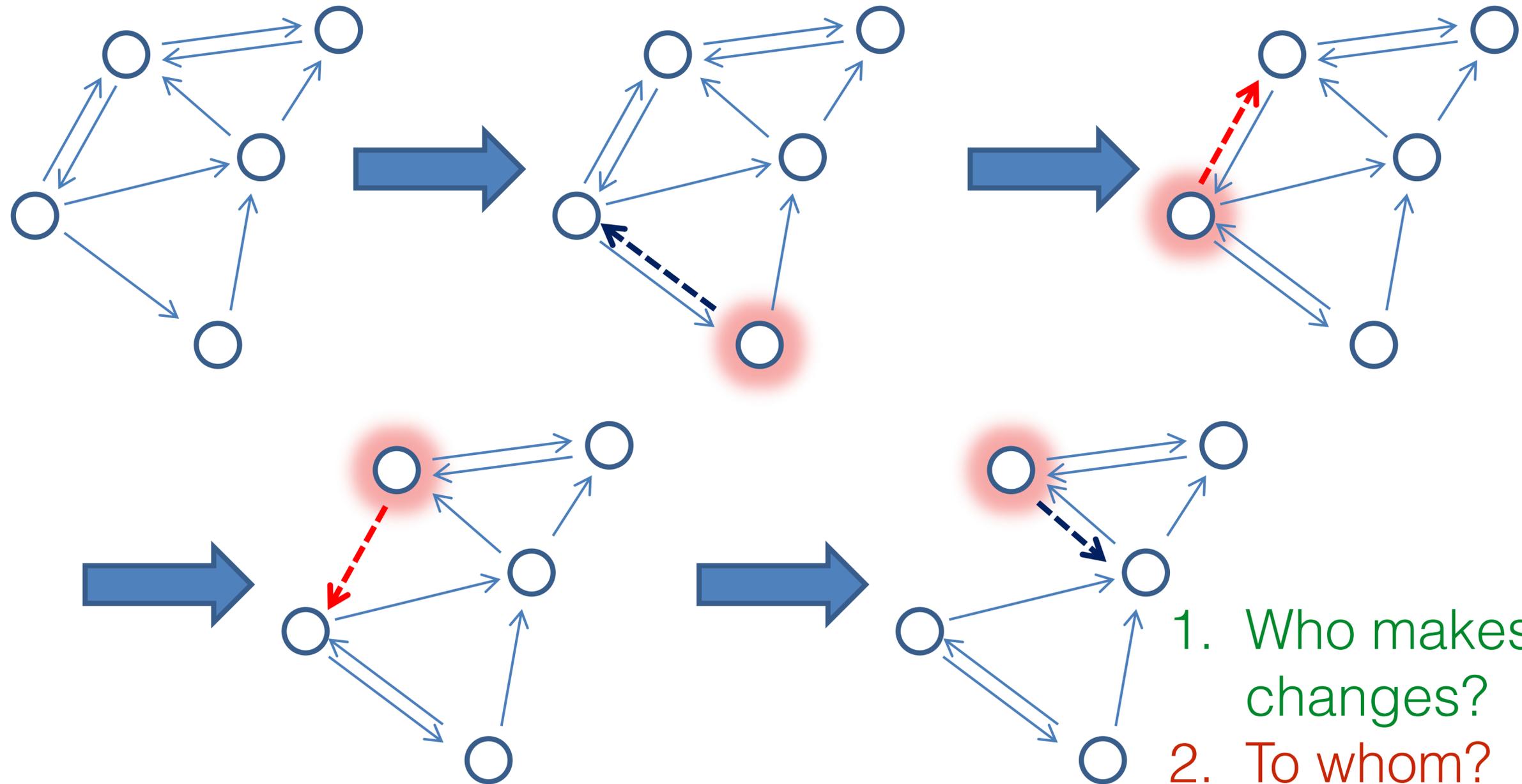
# Markov Assumption

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**The glowing actor does not remember the betrayal by the pink actor**

# Two Processes in Each Ministep



1. Who makes changes?
2. To whom?

*the*  
**Secret  
Sauce**

F1

F2

# The Two Functions

- Who gets a choice?

- This is the first part of the minstep
- A person (*ego* or the *focal actor*) is chosen to consider a change

## Rate Function

$$\lambda_i(x) = \exp \left( \sum_k \rho_k r_{ik}(x) \right)$$

- Who/what do they choose?

- Once an ego is chosen, we model which change she makes from her point of view
- In the case of a network tie, the candidates are people (*alters*)

## Evaluation Function

$$f_i(x) = \sum_k \beta_k s_{ik}(x)$$

# The Rate Function

$$\lambda_i(x) = \exp \left( \sum_k \rho_k r_{ik}(x) \right)$$

- Models how much change there is between  $t_1$  and  $t_2$
- Higher rates mean there is more change
  - More ministeps are necessary to provide actors with more opportunities to make more changes
- This can mean more ministeps than changes
  - Some actors, although given the opportunity to make a tie change, may decide they are actually satisfied
  - Some actors may revert earlier tie changes once their local neighbourhood changes again as a result of others' choices

# The Rate Function

$$\lambda_i(x) = \exp \left( \sum_k \rho_k r_{ik}(x) \right)$$

- Models how many opportunities each actor receives in a time period (between waves)
- Statistics  $r_{ik}(x)$  of  $i$ 's neighbourhood in  $x$  are weighted by parameters  $\rho_k$ 
  - These weights express whether actors in those configurations correlate with more ( $\rho_k > 0$ ) or less ( $\rho_k < 0$ ) change
- ((Technically,  $\lambda_i(x)$  is part of a (non-homogenous) Poisson process))
- Current studies typically assume a **periodwise constant rate**

# The Evaluation Function

$$f_i(x) = \sum_k \beta_k s_{ik}(x)$$

- Models the attractiveness of different network states  $x$  to actor  $i$  reachable within one step of the current network
- Statistics  $s_{ik}(x)$  of  $i$ 's neighbourhood in  $x$  are weighted by parameters  $\beta_k$
- These weights express whether such configurations are desired ( $\beta_k > 0$ ) or avoided ( $\beta_k < 0$ )

# The Evaluation Function

$$f_i(x) = \sum_k \beta_k s_{ik}(x)$$

- Models actors' choices
  - A value is calculated for each potential alter
  - The model: The alter that increases the evaluation function most is chosen
  - The estimation: Ties must have increased an evaluation function
- ((Technically,  $f_i(x)$  is part of a multinomial logit model for discrete, probabilistic choice))
- This is where the action is. It helps us answer whether we prefer happy friends or avoid depressed people

# Statistics and Effects

- By finding out how effects are weighted (the parameters), we can answer our research questions
- Each effect (“IV”) has an effect statistic which defines it
  - Are smokers popular?
    - Alter attribute effect:
$$s_i(x) = \sum_j x_{ij} v_j$$
  - Do students ethnically segregate?
    - Homophily effect:
$$s_i(x) = \sum_j x_{ij} I\{v_i = v_j\}$$
- They can depend on network configurations (i.e. the position of  $j$  in the network), or attributes (i.e. a characteristic of  $j$  or whether it is the same as  $i$ ), or both

# Covariates

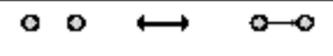
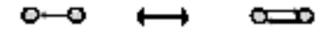
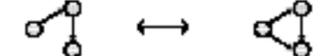
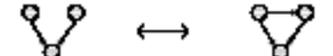
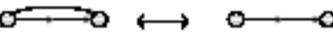
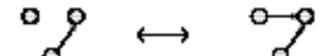
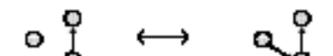
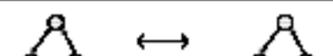
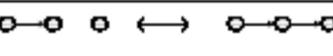
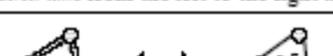
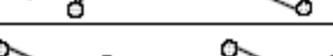
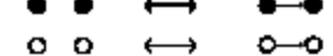
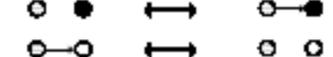
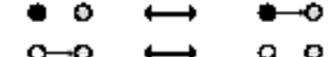
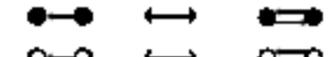
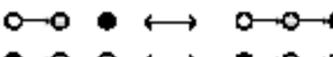
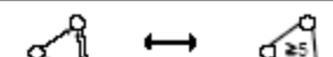
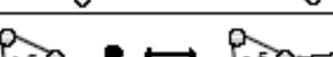
- Some effects rely on exogenous information
- There are four types:

| <b>Covariates</b> | <b>Monadic</b> | <b>Dyadic</b> |
|-------------------|----------------|---------------|
| <b>Constant</b>   | coCovar        | coDyadCovar   |
| <b>Changing</b>   | varCovar       | varDyadCovar  |

- For each type, multiple effects can be specified

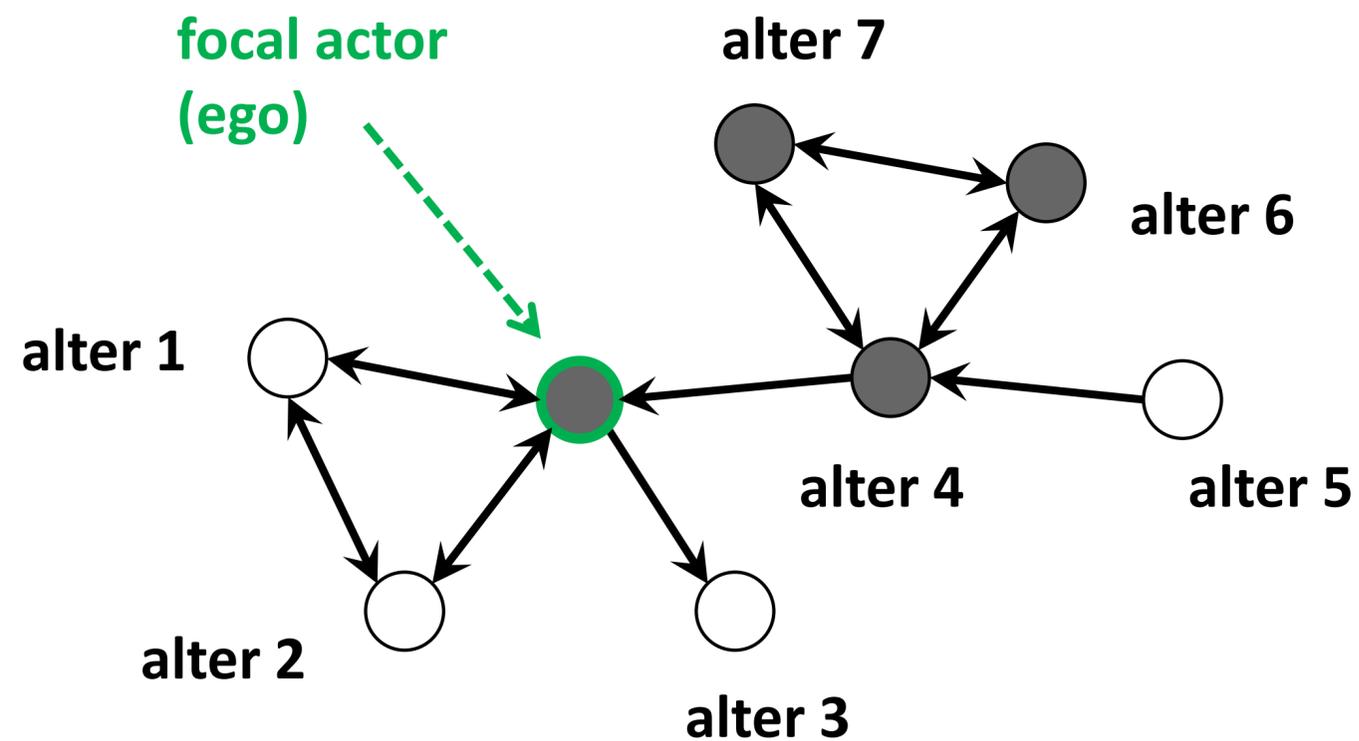
# Myriad Effects

TABLE 2  
SELECTION OF POSSIBLE EFFECTS FOR MODELING NETWORK EVOLUTION

| effect                        | network statistic   | effective transitions in network*  | verbal description   |
|-------------------------------|---|--|--|
| 1. outdegree                  | $x_{ij}$  |   | preference for ties to arbitrary others  |
| 2. reciprocity                | $x_{ij}x_{ji}$  |   | preference for reciprocated ties   |
| 3. transitive triplets        | $x_{ij} \sum_h x_{ih}x_{hj}$  |   | preference for being friend of the friends' friends  |
| 4. balance                    | $x_{ij} \text{strsim}_{ij}$   |   | preference for ties to structurally similar others   |
| 5. actors at distance two     | $\begin{cases} 1 & \text{if between}(h;ij) = 1 \text{ for some } h \\ 0 & \text{else} \end{cases}$  | <br>(the number of intermediaries is irrelevant)      | preference for keeping others at social distance two   |
| 6. popularity alter           | $x_{ij} \sum_h x_{hi}$  |   | preference for attaching to popular others, i.e., others who are often named as friend ('preferential attachment') |
| 7. activity alter             | $x_{ij} \sum_h x_{jh}$  |   | preference for attaching to active others, i.e., others who name many friends                                      |
| 8. 3-cycles                   | $x_{ij} \sum_h x_{jh}x_{hi}$  |   | preference for forming relationship cycles (negative indicator for hierarchical relations)                         |
| 9. betweenness                | $\sum_h \text{between}(i;hj)$   | <br>(no direct link from the left to the right actor) | preference for being in an intermediary position between unrelated others  |
| 10. dense triads              | $\sum_h \text{group}(ijh)$  |   | preference for being part of cohesive subgroups  |
| 11. peripheral                | $\sum_{hk} \text{peripheral}(i;jhk)$  |   | preference for unilaterally attaching to cohesive subgroups  |
| 12. similarity                | $x_{ij} \text{sim}_{ij}$  |   | preference for ties to similar others (selection)  |
| 13. behavior alter            | $x_{ij}z_i$   |   | main effect of alter's behavior on tie preference  |
| 14. behavior ego              | $x_{ij}z_j$   |   | main effect of ego's behavior on tie preference  |
| 15. similarity × reciprocity  | $x_{ij}x_{ji} \text{sim}_{ij}$  |   | preference for reciprocated ties to similar others   |
| 16. between dissimilar alters | $\sum_h (1 - \text{sim}_{jh}) \text{between}(i;jh)$   |   | preference for being in an intermediary position between unrelated, dissimilar others (brokerage potential)        |
| 17. similarity × dense triads | $\sum_h \text{group}(ijh)(\text{sim}_{ij} + \text{sim}_{ih})$                                       |   | preference for being part of behaviorally similar cohesive subgroups   |
| 18. behavior × peripheral     | $z_i \sum_{hk} \text{peripheral}(i;jhk)$  |   | behavior-specific preference for unilaterally attaching to cohesive subgroups                                      |
| 19. similarity × peripheral   | $\sum_{hk} (\text{peripheral}(i;jhk) \times (\text{sim}_{ij} + \text{sim}_{ih} + \text{sim}_{jk}))$ |   | preference for unilaterally attaching to behaviorally similar cohesive subgroups                                   |

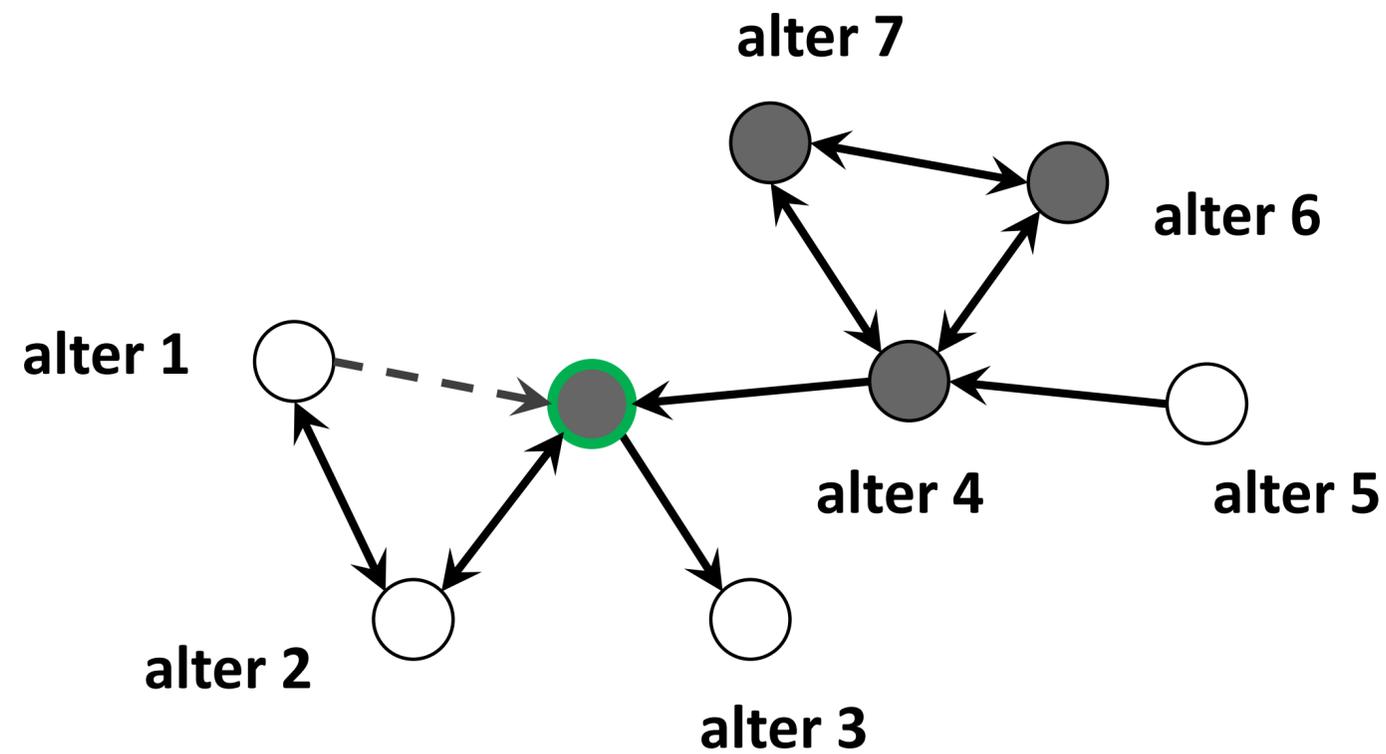
\* In the effective transitions illustrations, it is assumed that the behavioral dependent variable is dichotomous and centered at zero; the color coding is  $\circ$  = low score (negative),  $\bullet$  = high score (positive),  $\ominus$  = arbitrary score. The tie  $x_{ij}$  from actor  $i$  to actor  $j$  is the one that changes in the transition indicated by the double arrow. Illustrations are not exhaustive.

# Example of an actor's decision



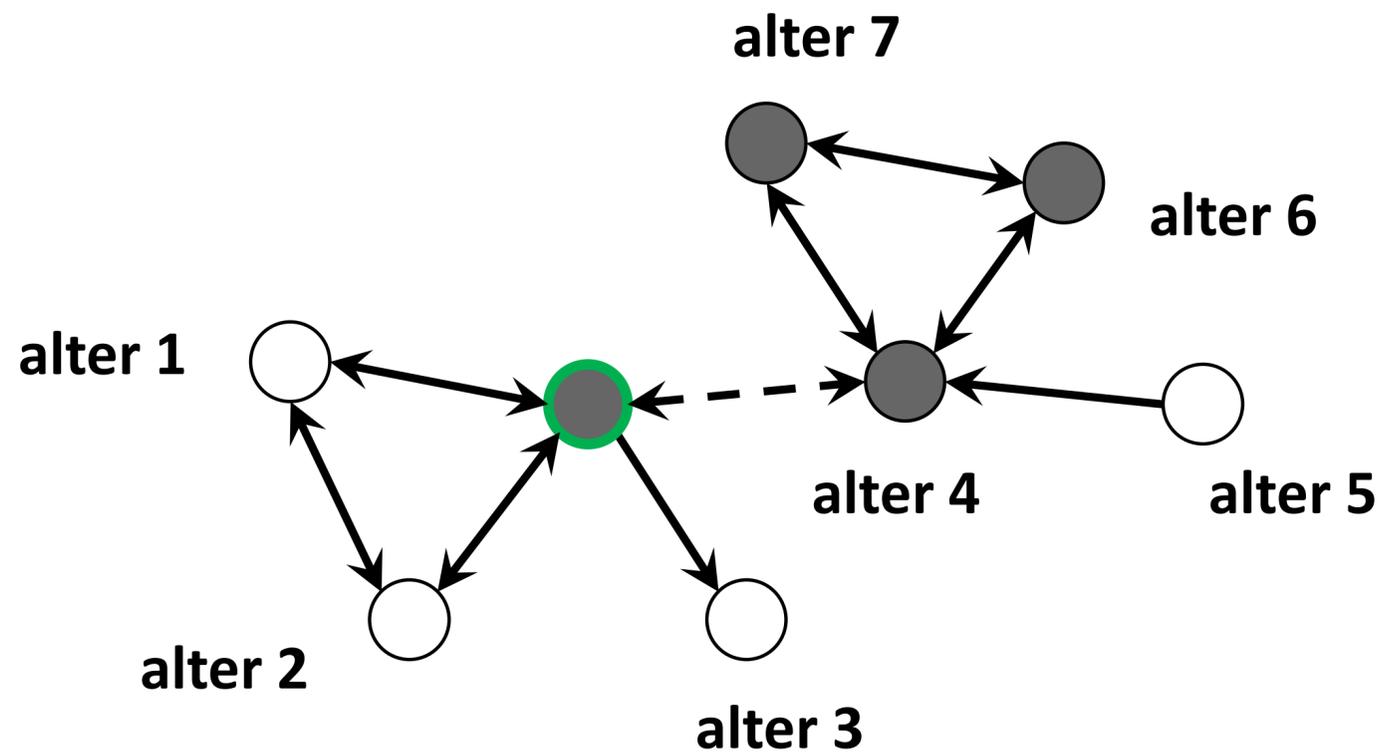
- Options
- drop tie to 1
- drop tie to 2
- drop tie to 3
- create tie to 4
- create tie to 5
- create tie to 6
- create tie to 7
- keep status quo

# Statistics for dropping tie to 1



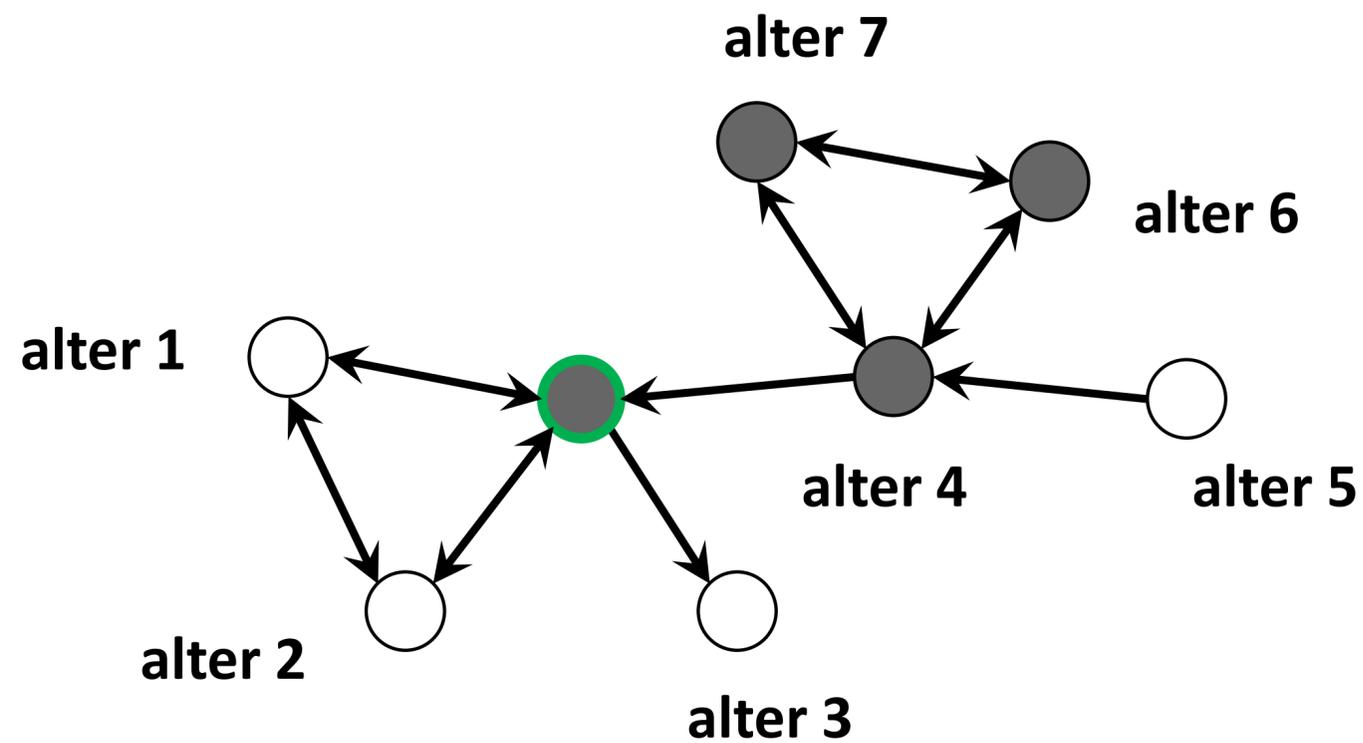
- 2 outgoing ties
- 1 reciprocated tie
- 0 transitive triplets
- 1 three-cycle
- 0 same colour

# Statistics for creating tie to 4



- 4 outgoing ties
- 3 reciprocated tie
- 2 transitive triplets
- 2 three-cycles
- 1 same colour

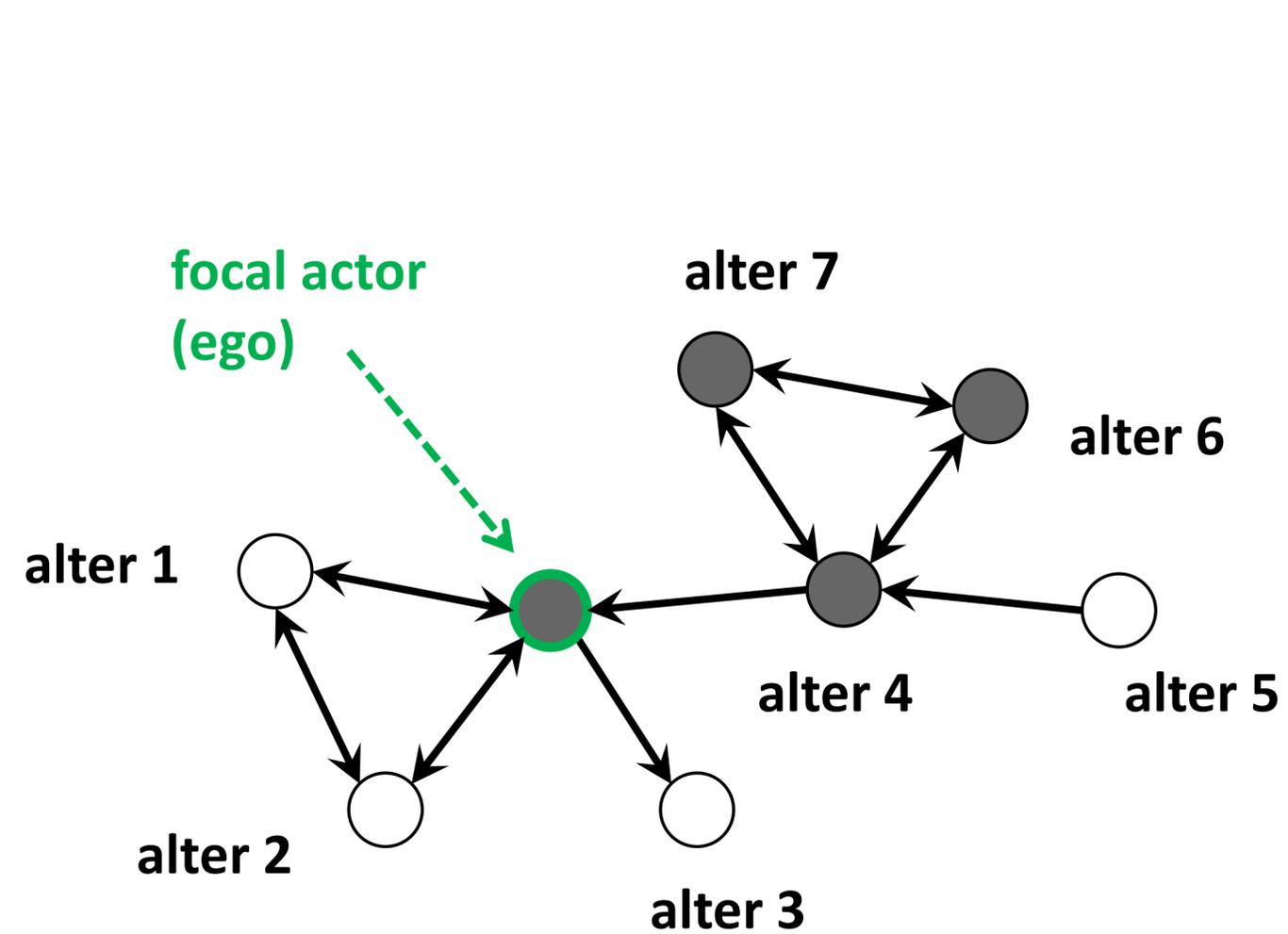
# Statistics for status quo



- 3 outgoing ties
- 2 reciprocated tie
- 2 transitive triplets
- 2 three-cycles
- 0 same colour

These calculations are done  
for all possible choices

# Statistics for all options



|            | #degree | #mutual | #trans | #3cycles | #same col. |
|------------|---------|---------|--------|----------|------------|
| Drop 1     | 2       | 1       | 0      | 1        | 0          |
| Drop 2     | 2       | 1       | 0      | 1        | 0          |
| Drop 3     | 2       | 2       | 2      | 2        | 0          |
| Create 4   | 4       | 3       | 2      | 2        | 1          |
| Create 5   | 4       | 2       | 2      | 3        | 0          |
| Create 6   | 4       | 2       | 2      | 3        | 1          |
| Create 7   | 4       | 2       | 2      | 3        | 1          |
| Status quo | 3       | 2       | 2      | 2        | 0          |

# Evaluating the options

$$f_i(x) = \sum_k \beta_k s_{ik}(x)$$

|                         |                          |      |                   | #degree | #mutual | #trans | #3cycles | #same col. |
|-------------------------|--------------------------|------|-------------------|---------|---------|--------|----------|------------|
| $f_i(\text{drop1})$     |                          |      |                   |         |         |        |          |            |
| $f_i(\text{drop2})$     | $\beta_{\text{degree}}$  | -2.6 | <b>Drop 1</b>     | 2       | 1       | 0      | 1        | 0          |
| $f_i(\text{drop3})$     | $\beta_{\text{mutual}}$  | 1.8  | <b>Drop 2</b>     | 2       | 1       | 0      | 1        | 0          |
| $f_i(\text{create4})$   |                          |      | <b>Drop 3</b>     | 2       | 2       | 2      | 2        | 0          |
| $f_i(\text{create5})$   | $\beta_{\text{trans}}$   | 0.4  | <b>Create 4</b>   | 4       | 3       | 2      | 2        | 1          |
| $f_i(\text{create6})$   | $\beta_{\text{3cycles}}$ | -0.7 | <b>Create 5</b>   | 4       | 2       | 2      | 3        | 0          |
| $f_i(\text{create7})$   |                          |      | <b>Create 6</b>   | 4       | 2       | 2      | 3        | 1          |
| $f_i(\text{statusquo})$ | $\beta_{\text{same}}$    | 0.8  | <b>Create 7</b>   | 4       | 2       | 2      | 3        | 1          |
|                         |                          |      | <b>Status quo</b> | 3       | 2       | 2      | 2        | 0          |

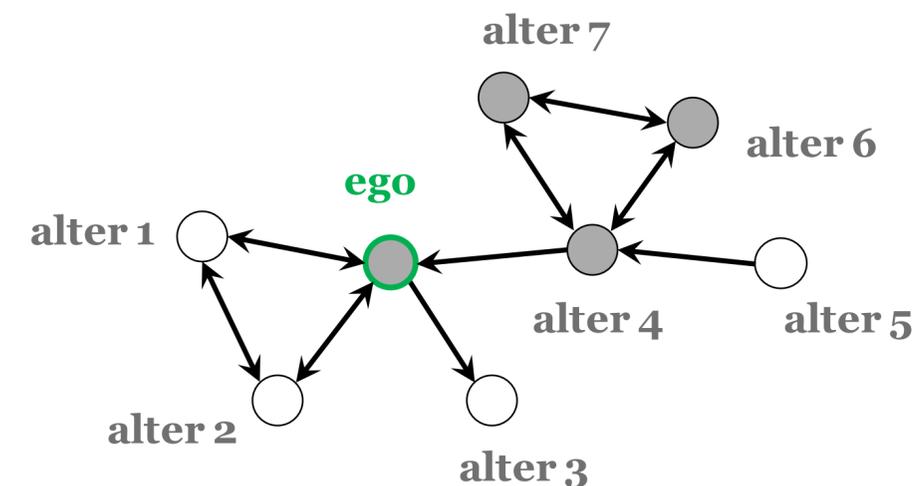
# Transforming to probabilities

Using underlying multinomial:

$$p_{i \rightsquigarrow j}(x, \beta) = \frac{\exp(f(x^{i \rightsquigarrow j}, \beta))}{\sum_{k=1}^n \exp(f(x^{i \rightsquigarrow k}, \beta))}$$

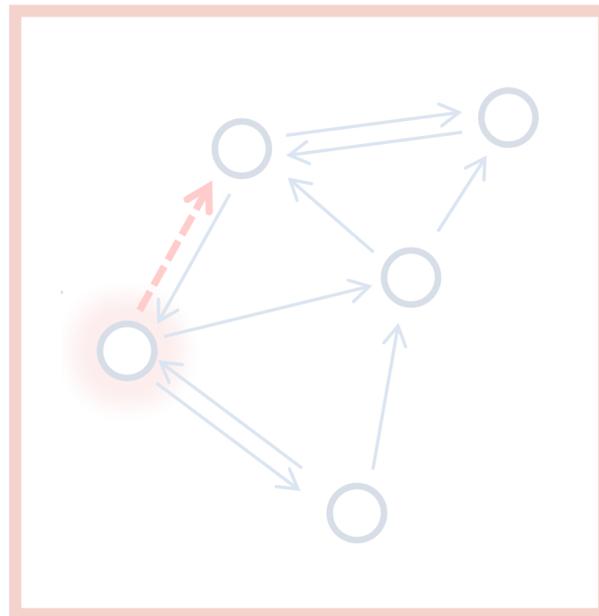
|                   | Evaluation | Exponent. | Prob. |
|-------------------|------------|-----------|-------|
| <b>Drop 1</b>     | -4.1       | 0.017     | 10%   |
| <b>Drop 2</b>     | -4.1       | 0.017     | 10%   |
| <b>Drop 3</b>     | -2.2       | 0.111     | 68%   |
| <b>Create 4</b>   | -4.8       | 0.008     | 5%    |
| <b>Create 5</b>   | -8.1       | 0.000     | 0%    |
| <b>Create 6</b>   | -7.3       | 0.001     | 1%    |
| <b>Create 7</b>   | -7.3       | 0.001     | 1%    |
| <b>Status quo</b> | -4.8       | 0.008     | 5%    |

Dropping tie to alter 3 is the most likely choice for ego

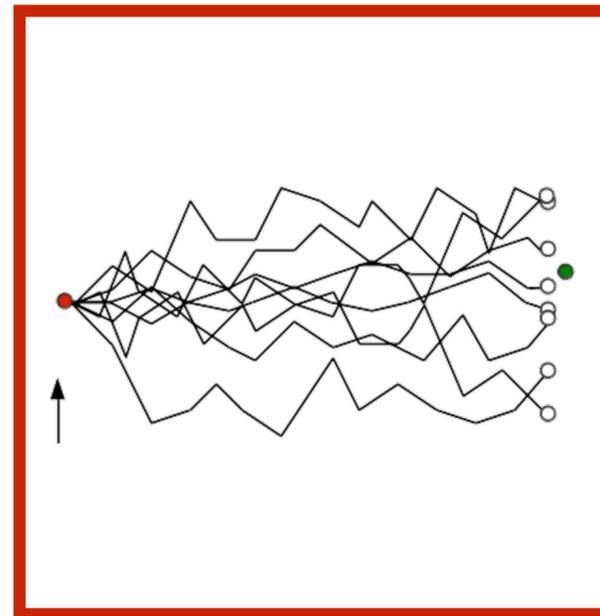


# SAOM

Model



Estimation



Influence

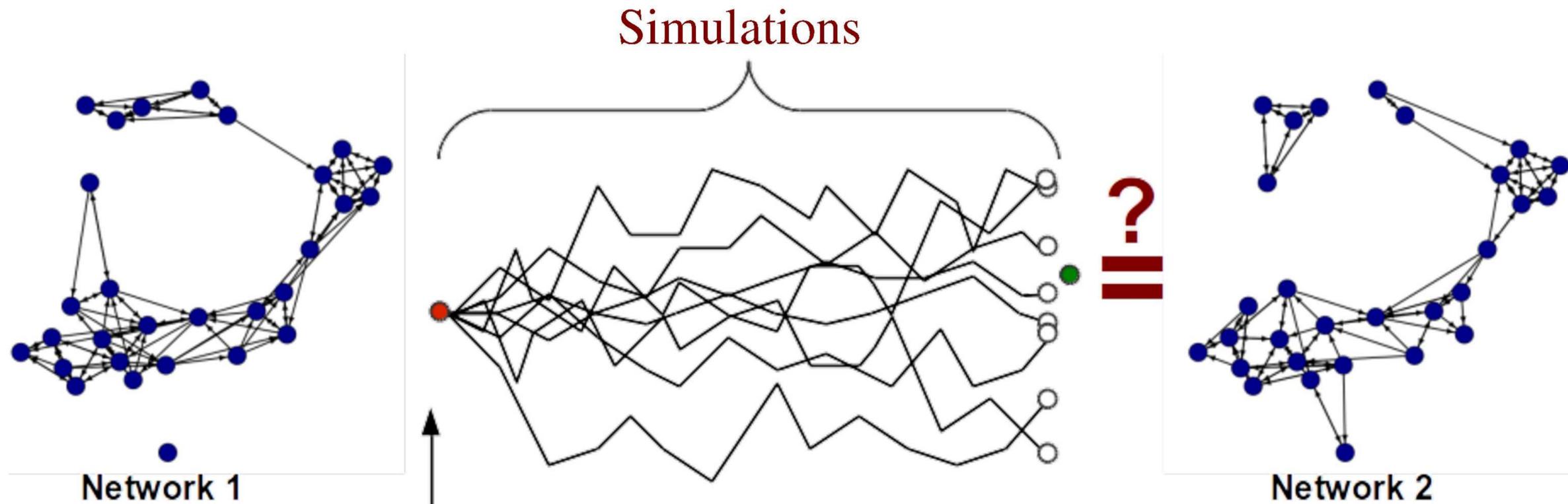


# Estimation

- So we now have a well-defined probability model, from which we can simulate networks using defined parameters ( $\beta$ )
- But what we usually want to do is *estimate* parameters from *observed* data!
- We do this using [RSiena](#) (“SIENA” = Simulation Investigation for Empirical Network Analysis)



# SIENA estimates SAOMs through simulations



adjust parameters **no** Are the simulated networks similar to network 2?

**yes**

The parameters are "good" descriptors of the social processes shaping network 2

# Three Estimation Methods

- **Method of Moments (MoM)**

- Take the network at the first time point and simulate a certain number of mini-steps with some initial  $\beta$  values
- Compare the simulated networks to the observed network at the second time point
- According to the differences between observed and simulated networks, we update the  $\beta$  values
- Rinse and repeat until the simulated networks “closely” resemble the observed one

- **Maximum Likelihood (ML)**

- Actually connects two observations by chains of ministeps and estimates parameters from these chains

- **Bayesian (Bayes)**

- For multilevel analysis of networks and enthusiasts

# Estimation Results

- While the model is more complicated, RSiena spits out a table at the end, the second part of which can be interpreted like that of a multinomial regression
- Each parameter estimate has a standard error
- If the  $t$ -ratio ( $= \beta/se$ )  $\geq 2$ , then we can say that we can reject the null hypothesis of there being no effect

|   | Model 1          | Model 3          |
|---|------------------|------------------|
| <b><i>Rate function friendship</i></b>      |                  |                  |
| Rate of change $t_1 \rightarrow t_2$        | 7,54 (0,97)      | 10,87 (2,63)     |
| Rate of change $t_2 \rightarrow t_3$        | 2,73 (0,45)      | 3,04 (0,52)      |
| Rate of change $t_3 \rightarrow t_4$        | 3,29 (0,49)      | 3,80 (0,65)      |
| <b><i>Objective function friendship</i></b> |                  |                  |
| Outdegree                                   | -1,92 (0,17) *** | -2,19 (0,16) *** |
| Reciprocity                                 | —                | 0,84 (0,17) ***  |
| Transitive triplets                         | —                | 0,18 (0,03) ***  |
| primary school friendship                   | 0,54 (0,21) *    | 0,40 (0,20) *    |
| Male alter                                  | 0,30 (0,18)      | 0,05 (0,17)      |
| Male ego                                    | 0,11 (0,19)      | -0,17 (0,18)     |
| Same sex                                    | 1,70 (0,18) ***  | 0,93 (0,18) ***  |

*strongly  
biased*

# Model Specification

- Researchers usually come with *theory* or at least *hypotheses*
- SAOMs are not for exploration
- Beware spuriousness...
  - Attribute vs centrality (popularity)
  - Homophily vs cohesion (reciprocity, transitivity)

|   | Model 1          | Model 3          |
|---|------------------|------------------|
| <b><i>Rate function friendship</i></b>      |                  |                  |
| Rate of change $t_1 \rightarrow t_2$        | 7,54 (0,97)      | 10,87 (2,63)     |
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| Same sex                                    | 1,70 (0,18) ***  | 0,93 (0,18) ***  |

*strongly  
biased*

# Parameter Interpretation

- Estimated parameters need to be interpreted as within ministeps
- So we interpret the parameters as: when a chosen ego  $i$  is faced with a decision to form a tie to either of two alters,  $j_1$  or  $j_2$ , that differ only on one statistic value, then the odds ratio is as follows:

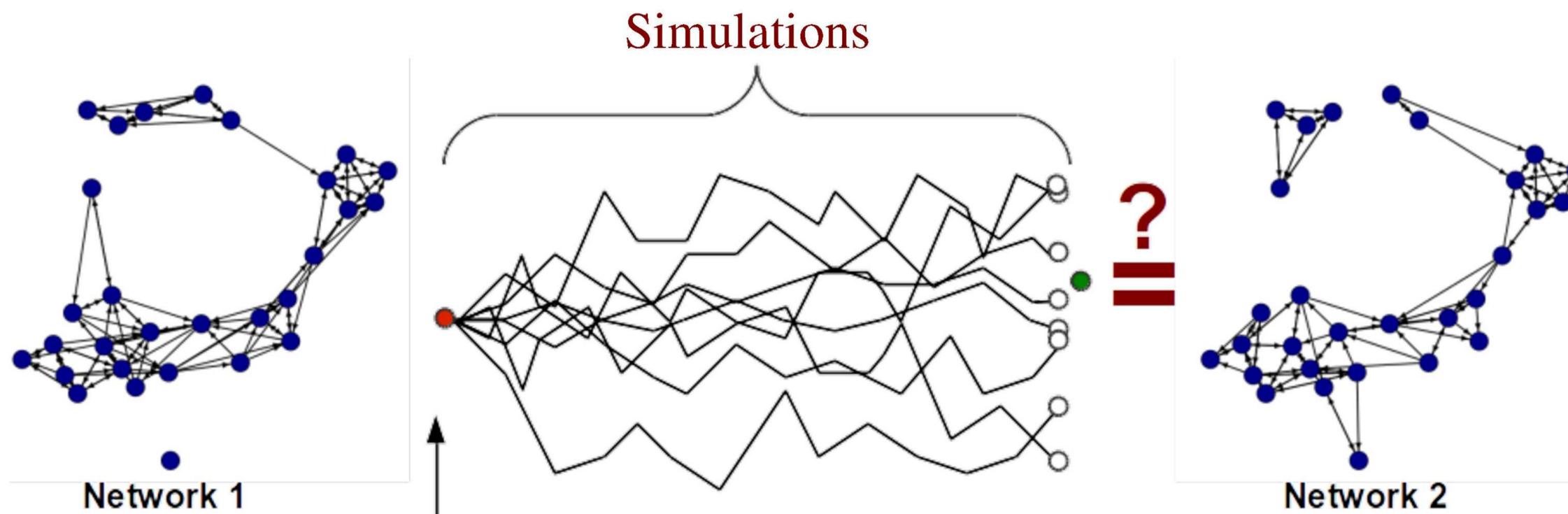
$$\frac{p_{i \rightsquigarrow j_1}}{p_{i \rightsquigarrow j_2}} = \frac{\exp(f(x^{i \rightsquigarrow j_1}, \beta))}{\exp(f(x^{i \rightsquigarrow j_2}, \beta))} = \frac{\exp(\beta s_{j_1})}{\exp(\beta s_{j_2})}$$

- So, say  $i$  can send a tie to  $j_1$  or  $j_2$ , which only differ in that  $j_1$  sends a tie to  $i$  and  $j_2$  does not, then given a reciprocity parameter of 2,  $\exp(2 \times 1) / \exp(2 \times 0) = 7.39$
- $i$  is 7.39 times more likely to send a tie to  $j_1$  than  $j_2$



Diagnostics

# But what does “good” mean?



Are the simulated networks  
adjust parameters **no** similar to network 2?

**yes**

The parameters are "good" descriptors of  
the social processes shaping network 2

# Target statistics $Z$ are listed in the SIENA output file

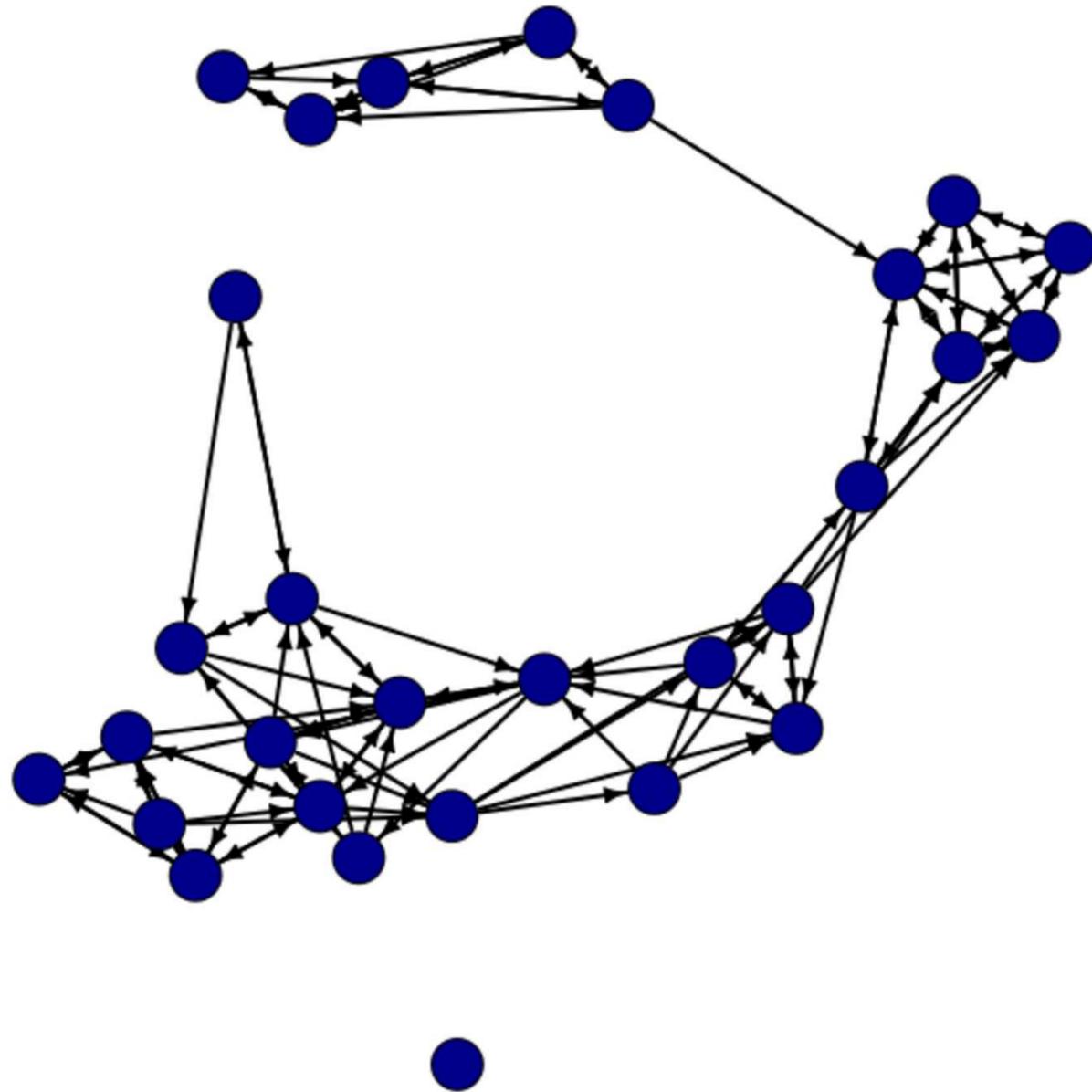
Observed values of target statistics are

|                                       |          |
|---------------------------------------|----------|
| 1. Number of ties                     | 99.0000  |
| 2. Number of reciprocated ties        | 72.0000  |
| 3. Number of transitive triplets      | 164.0000 |
| 4. 3-cycles                           | 47.0000  |
| 5. Sum of squared indegrees           | 403.0000 |
| 6. Same values on coo.coCovar         | 47.0000  |
| 7. Sum of indegrees x gender.coCovar  | -5.0345  |
| 8. Sum of outdegrees x gender.coCovar | -4.0345  |
| 9. Same values on gender.coCovar      | 90.0000  |

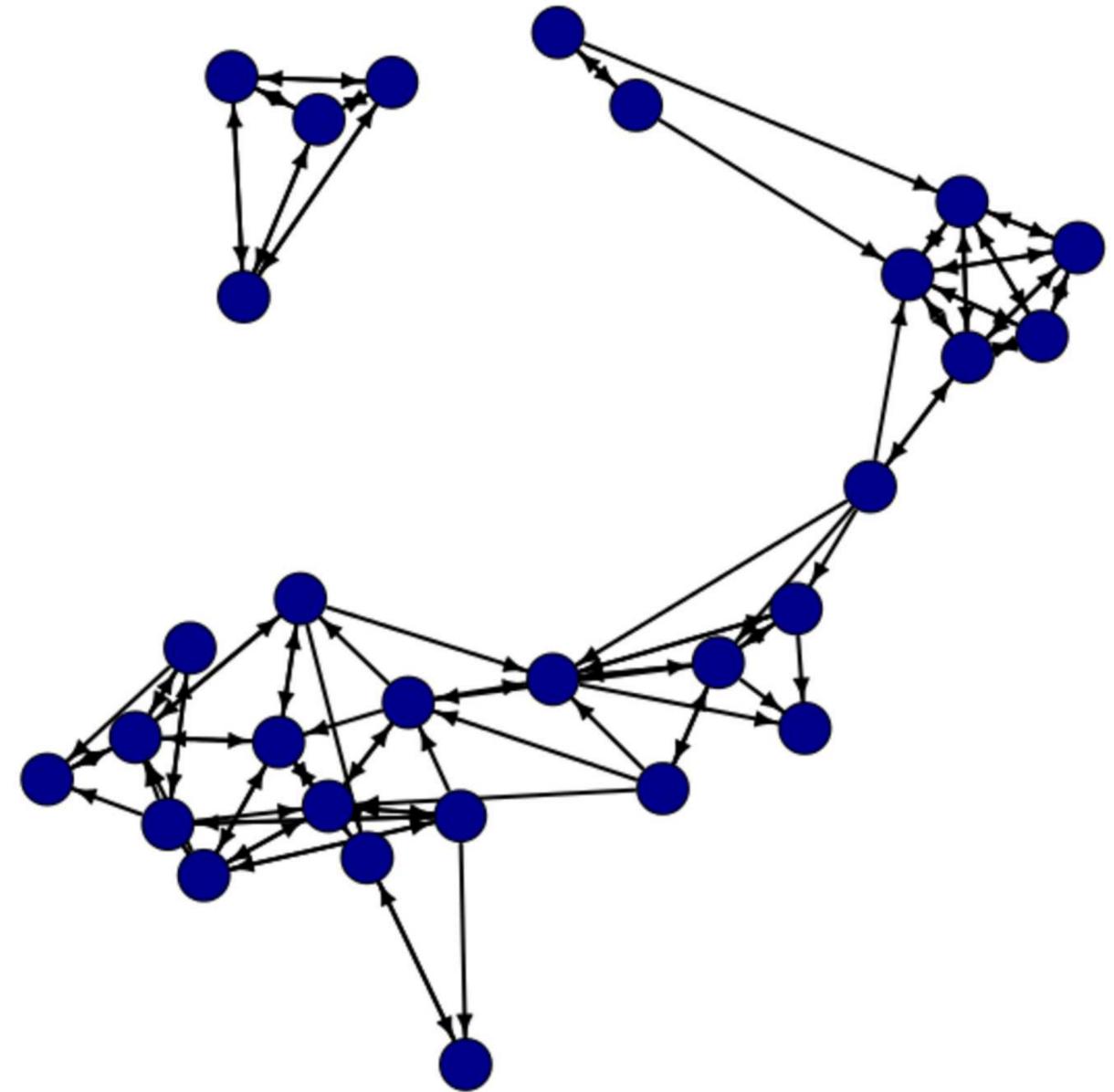
- **MoM** aims at creating networks that have statistics close to the ones above
- More formally, parameters  $\theta = \{\alpha, \beta\}$  that generate networks for which  $E_{\theta} = \{Z\}$  and are stable have **converged**
- But do these simulated networks resemble other, non-modelled macro features of the network such as the degree distribution, the triad census, etc? (i.e. **goodness of fit**)

# So, which forces shape this social network's evolution?

Network wave 1

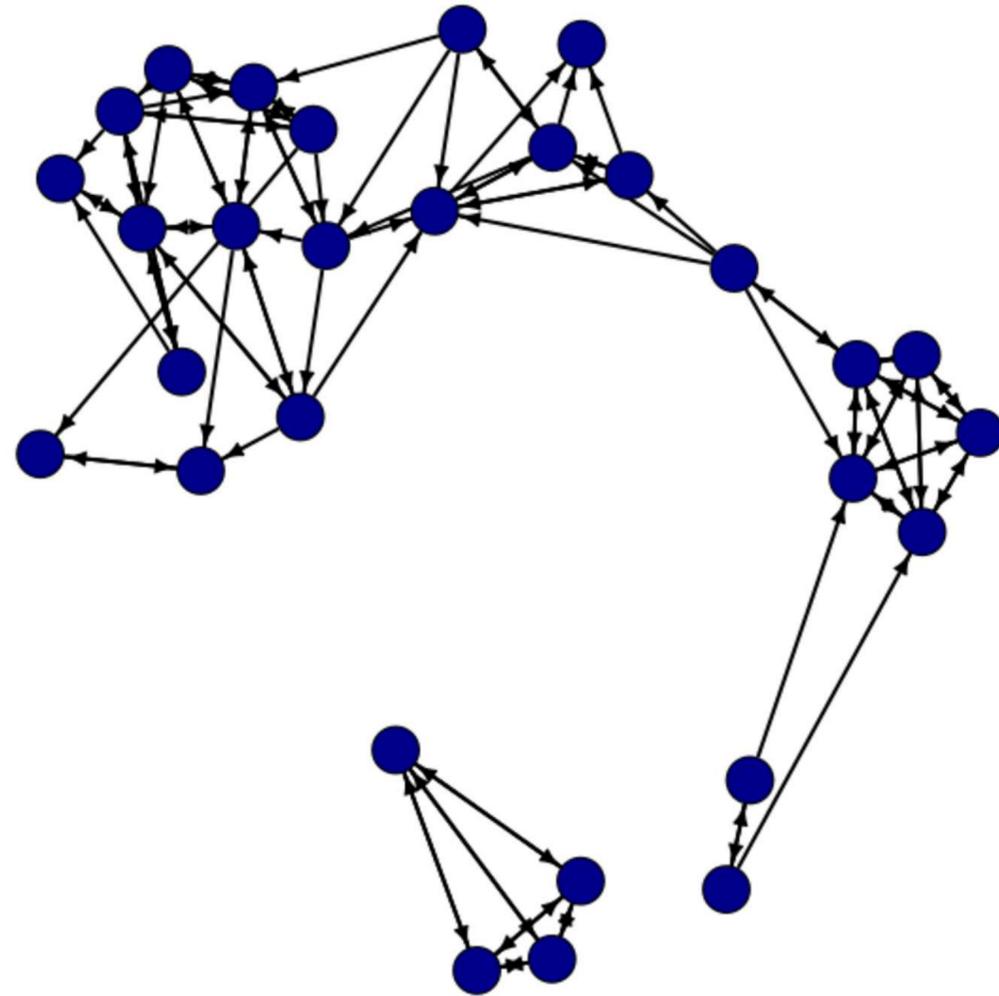


Network wave 2

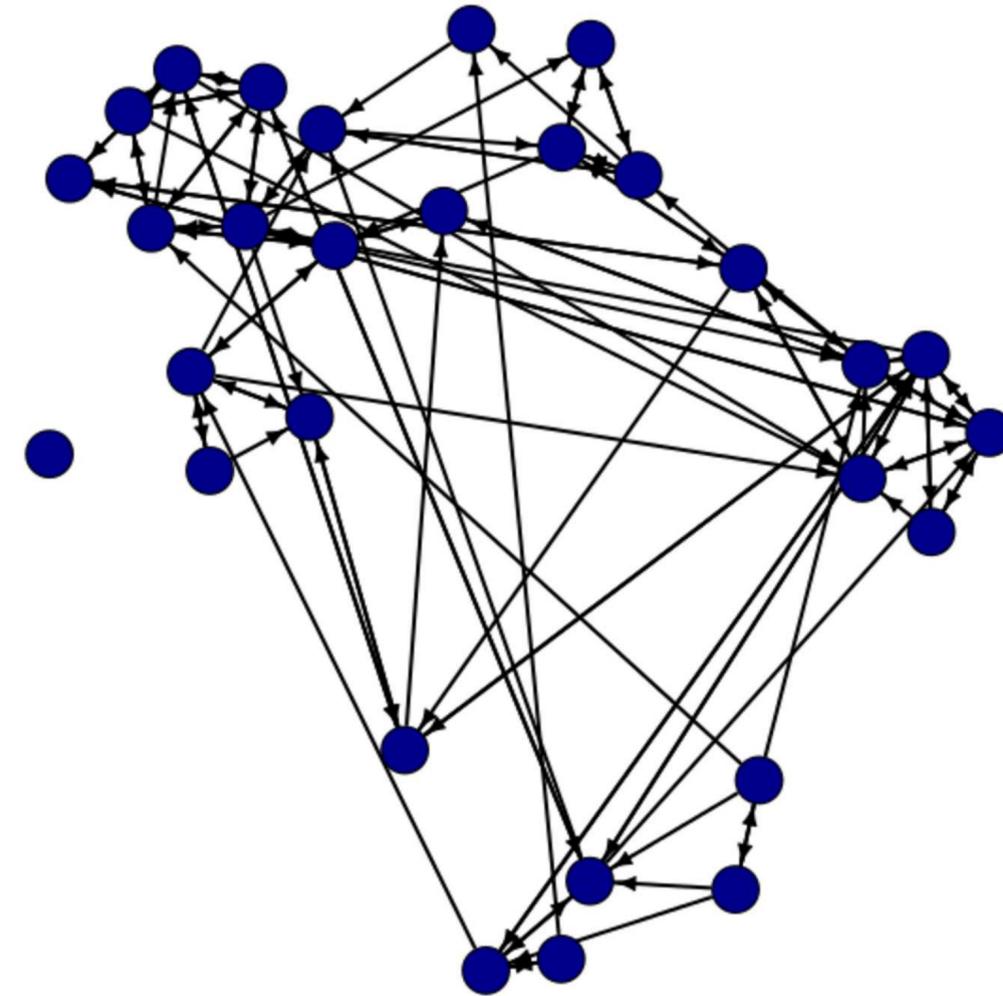


# Degree + Reciprocity

Network wave 2



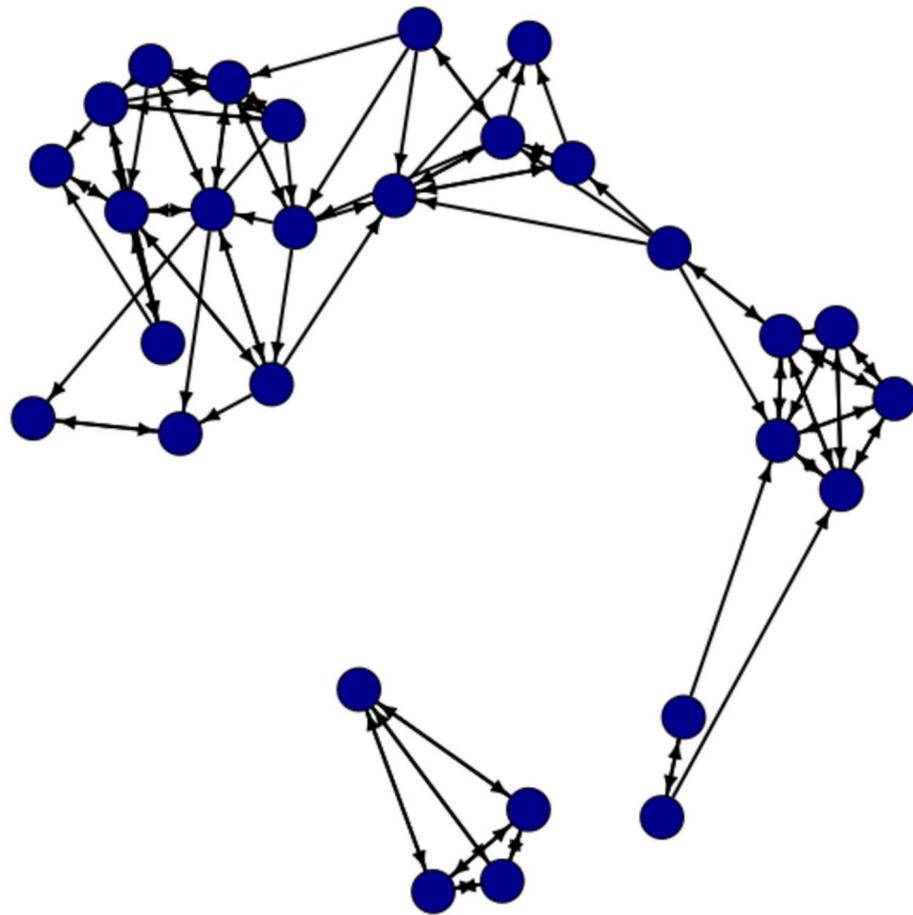
Simulated network



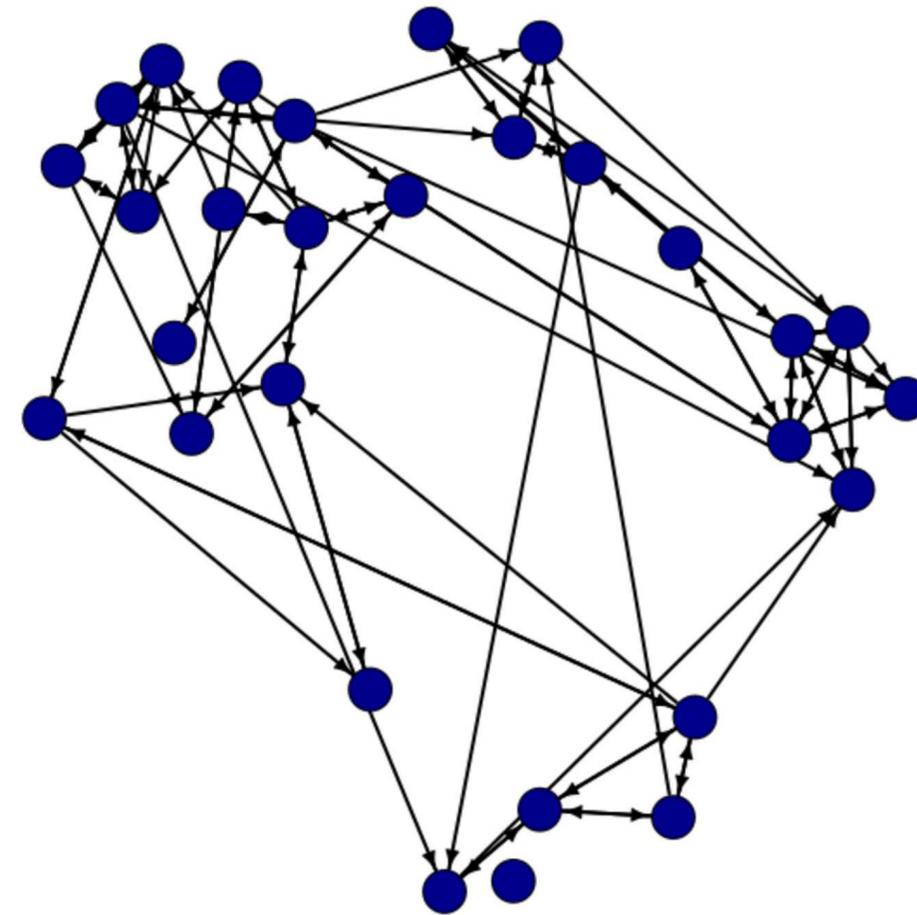
- While the model has converged and the two parameters are highly significant, the model does not represent groups very well...

# + Transitivity and 3-Cycles

Network wave 2

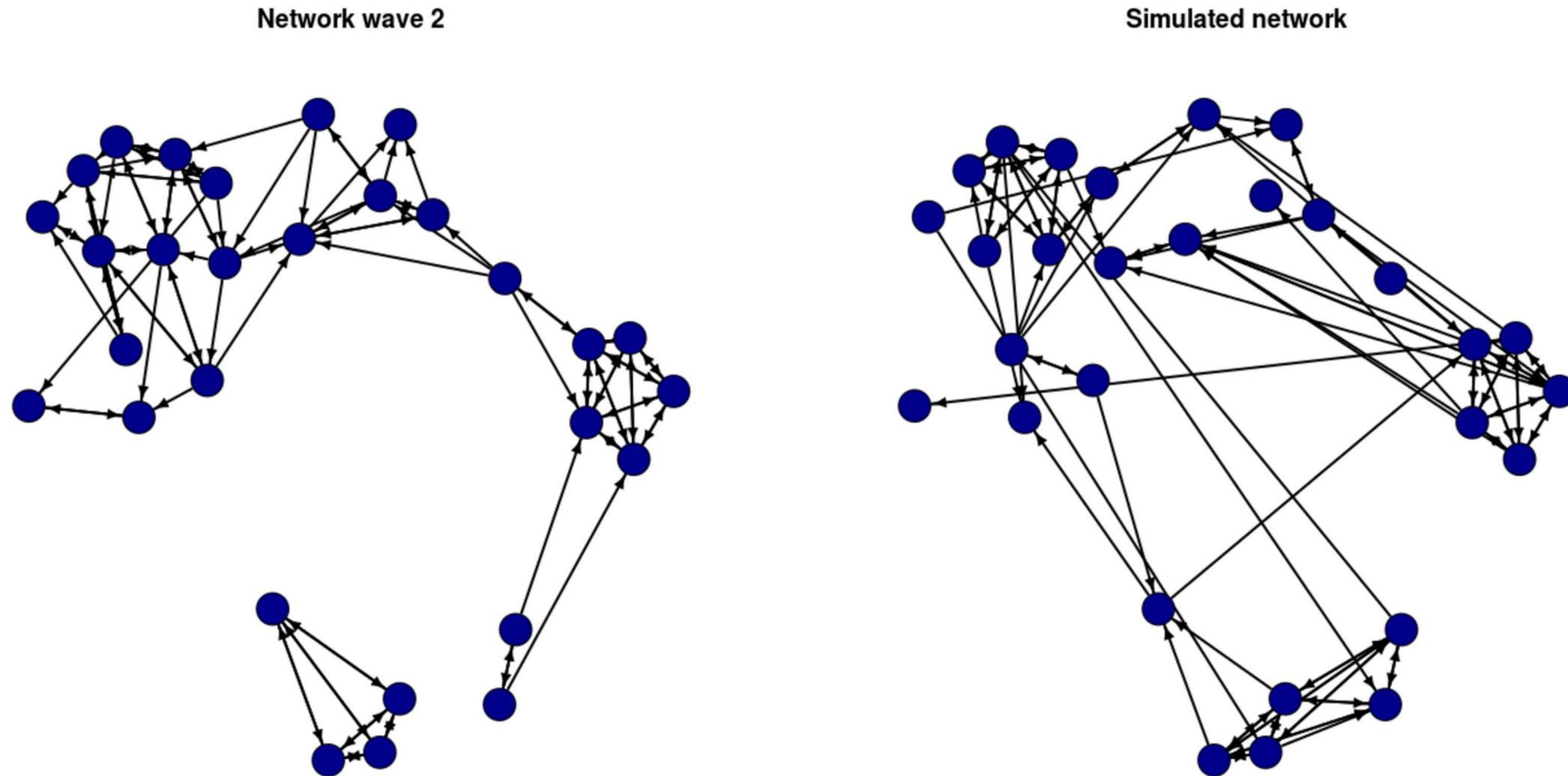


Simulated network



- Group boundaries are clearer but there are still too many connections between groups

# + Gender Homophily

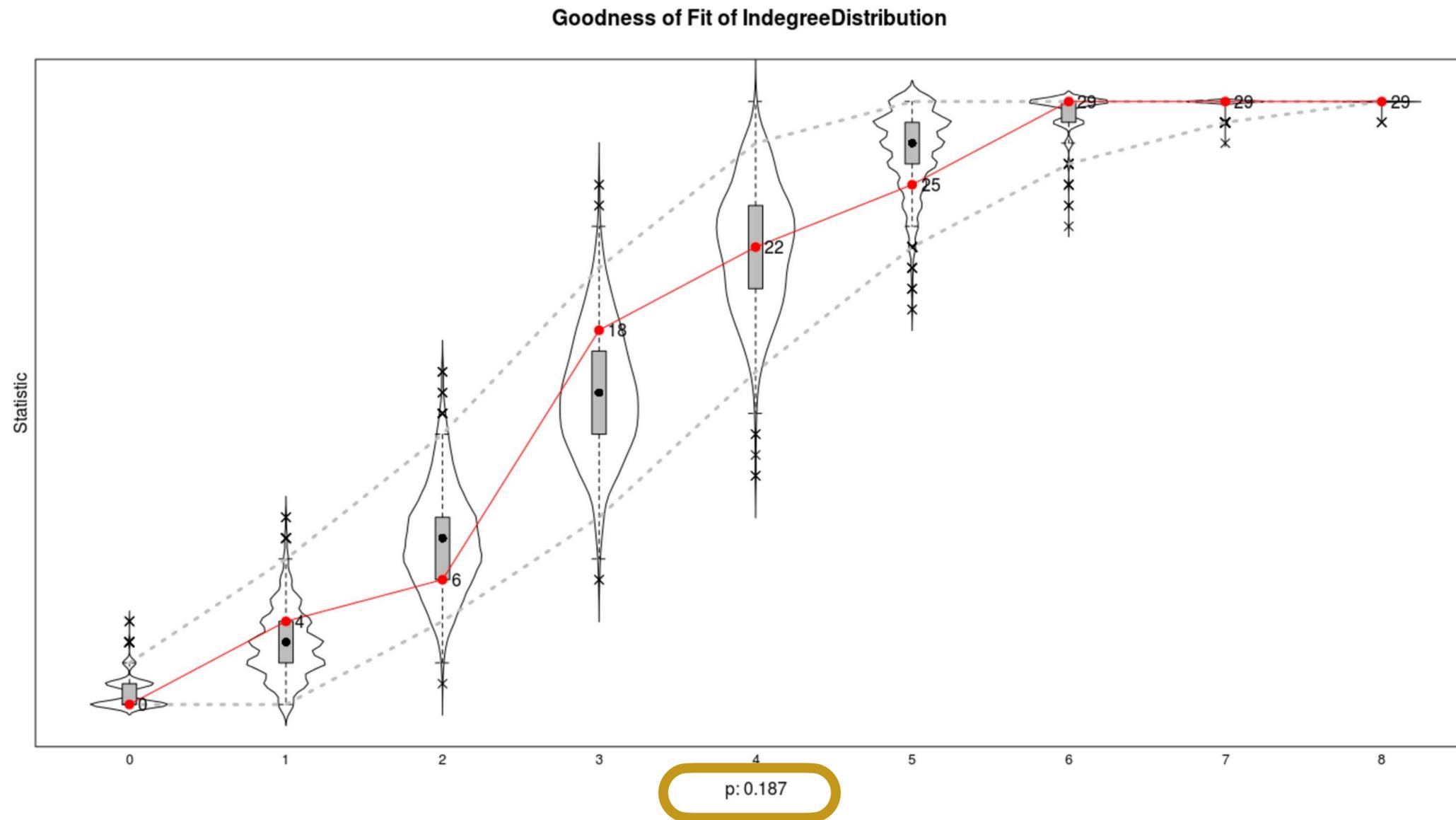


- Fewer links between groups of different gender but still many between-group ties within a gender
- One could try further structural and attribute-related effects

# sienaGOF() does this comparison more systematically

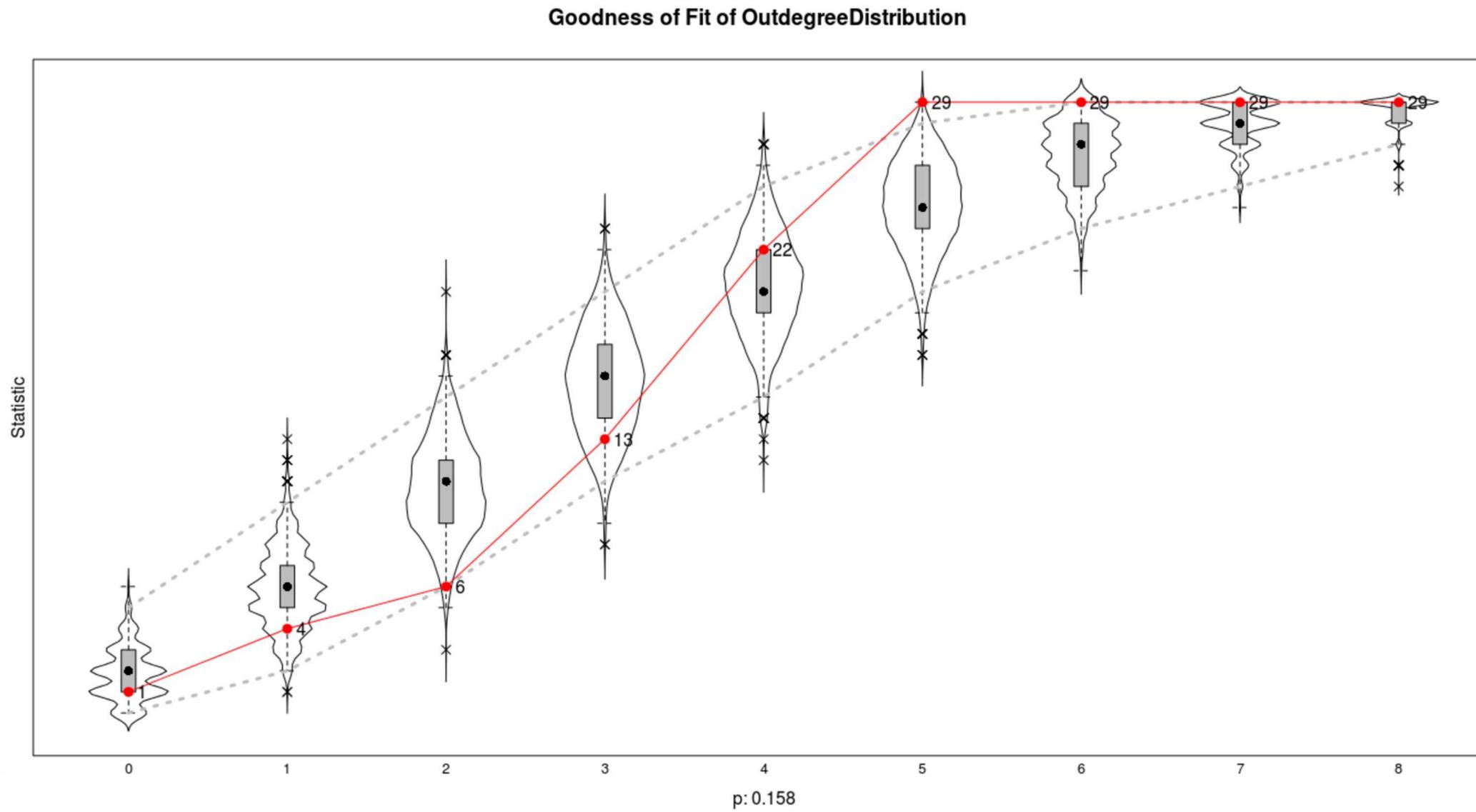
- sienaGOF() tests particular macro features of the simulated social networks and compares them to the empirically observed networks
  - Degree distribution
  - Geodesic distances
  - Triad census
- sienaGOF() takes all simulated networks into account, as opposed to the visual inspection, where we only looked at one

# Indegree GOF



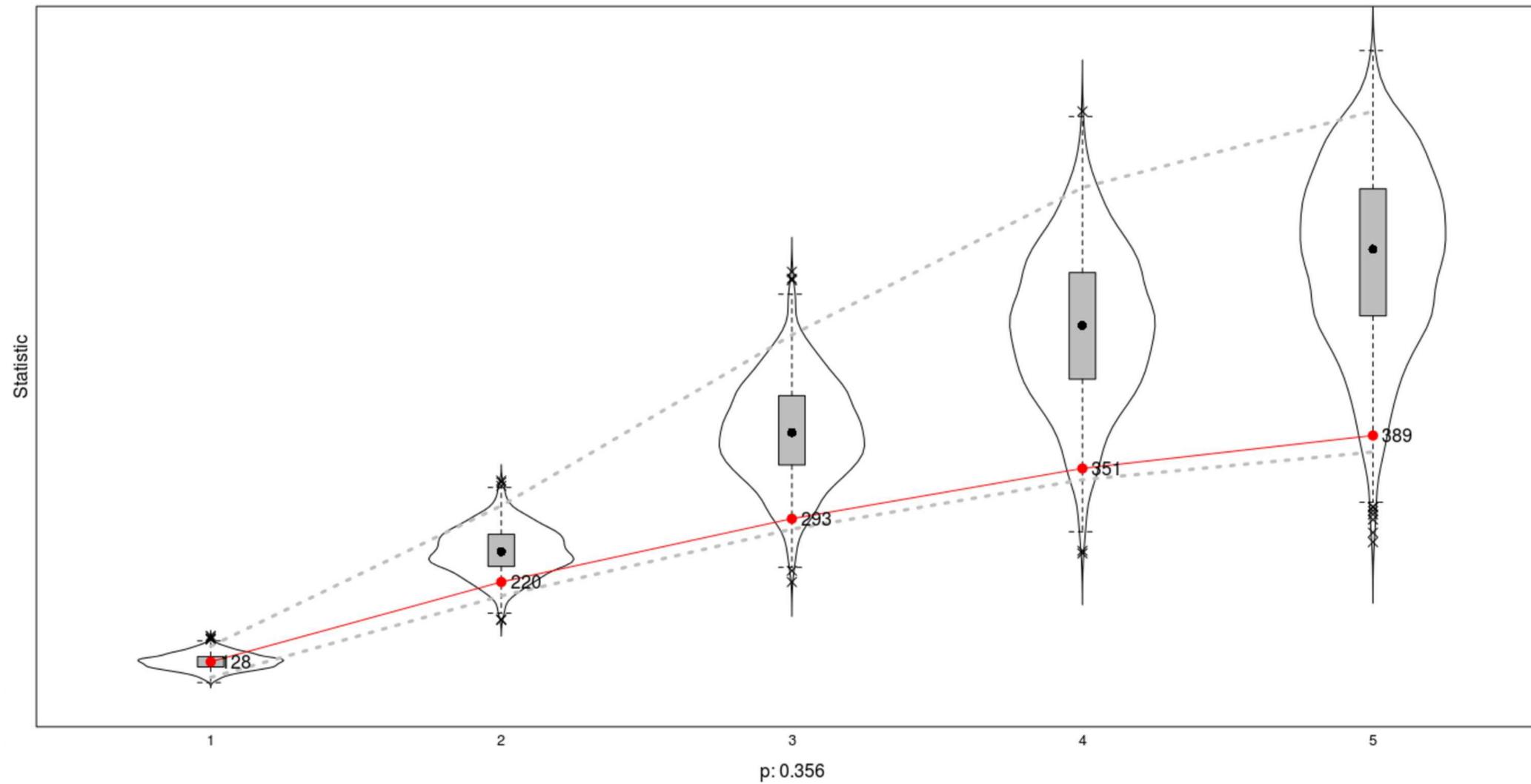
p-value *over* .05 suggests reasonable fit

# Outdegree GOF



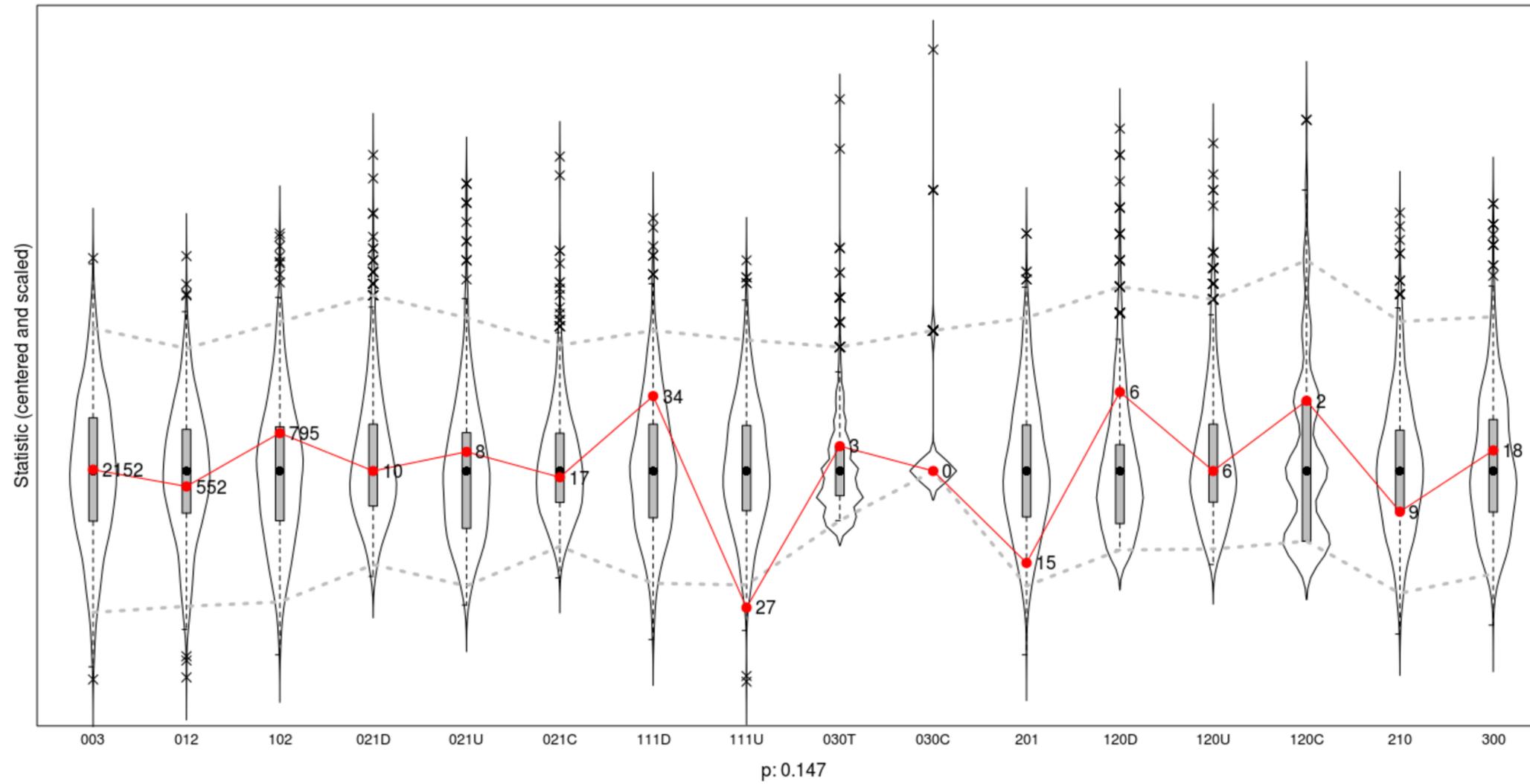
# Geodesic GOF

Goodness of Fit of GeodesicDistribution



# Triad census GOF

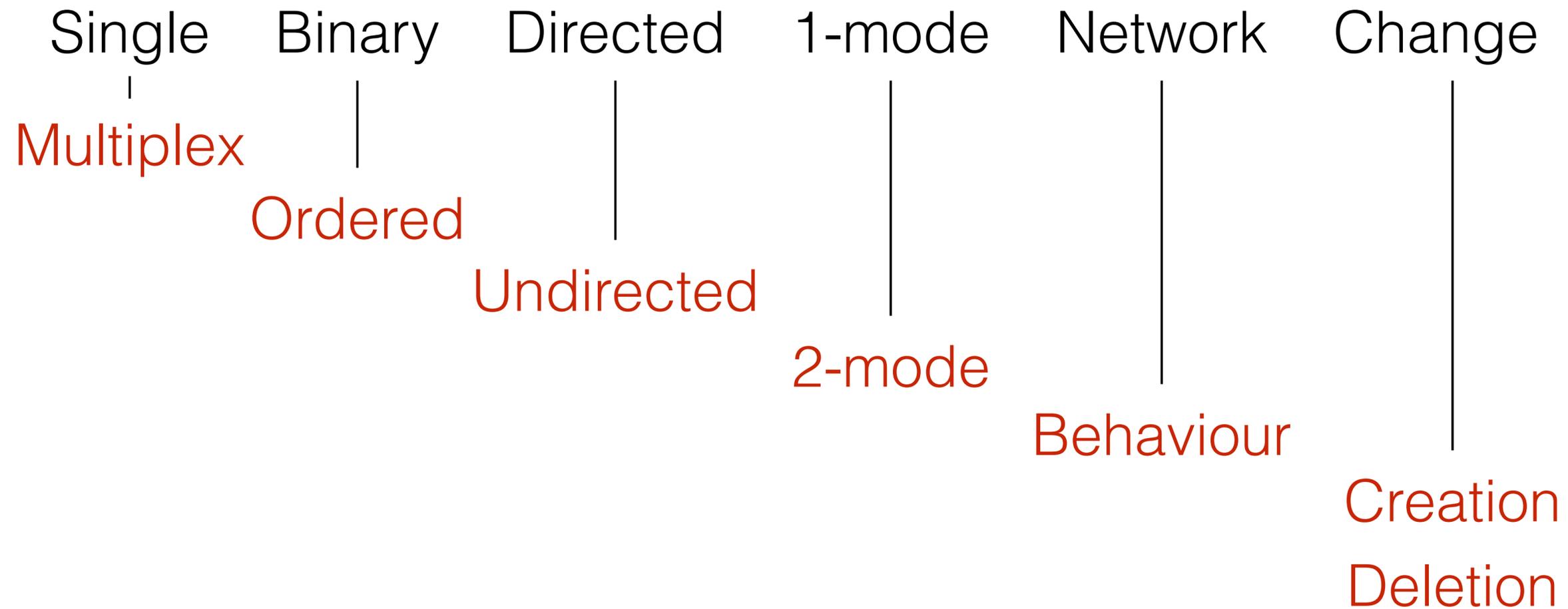
Goodness of Fit of TriadCensus



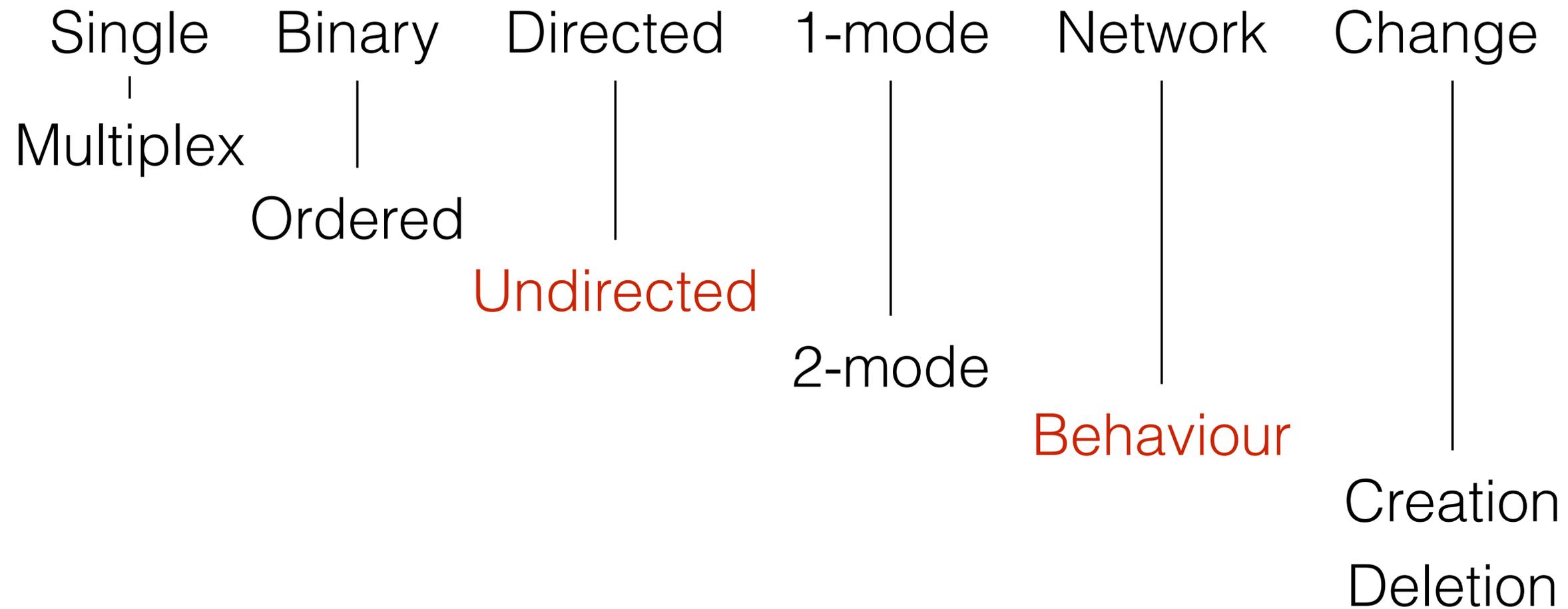
# Standard Model

Single Binary Directed 1-mode Network Change

# Model Extensions

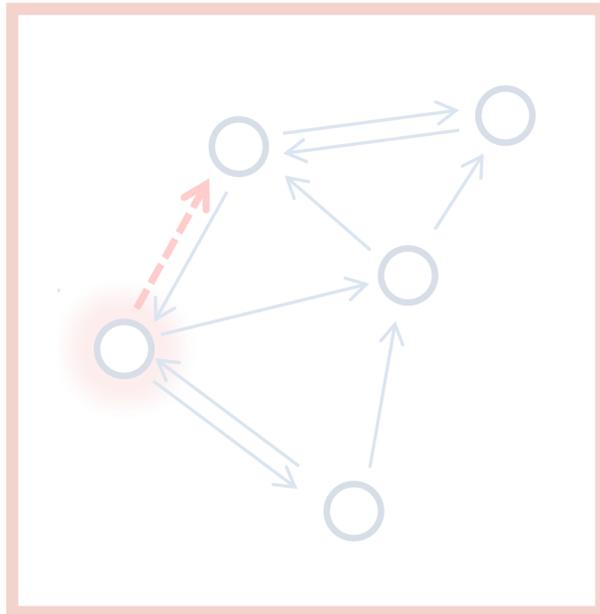


# Model Extensions

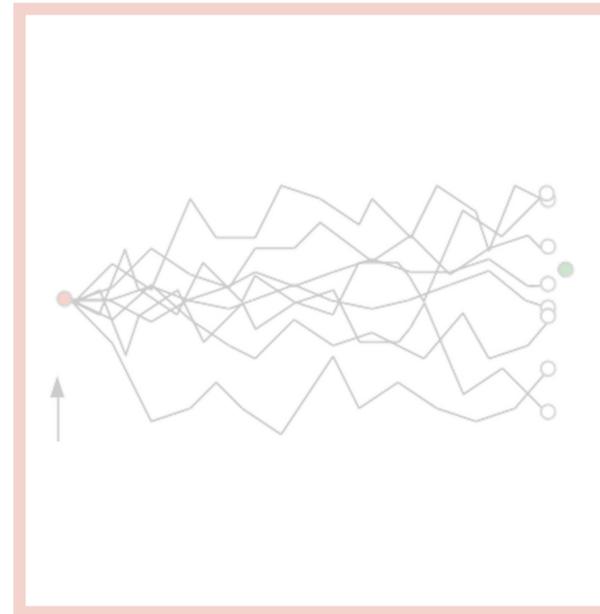


# SAOM

Model



Estimation



Influence





A mother's view  
of social  
influence:  
“if your friend  
jumped off a  
bridge, would  
you?”

# An example from my childhood friend Zak...

- **Manifest homophily**: Zak and I are friends because we both jump off bridges

## Selection

- **Secondary homophily, observable**: Zak and I are friends because we are in the same travelling and thrill-seeking club

- **Latent homophily, unobservable**: Zak and I both like going on rollercoasters

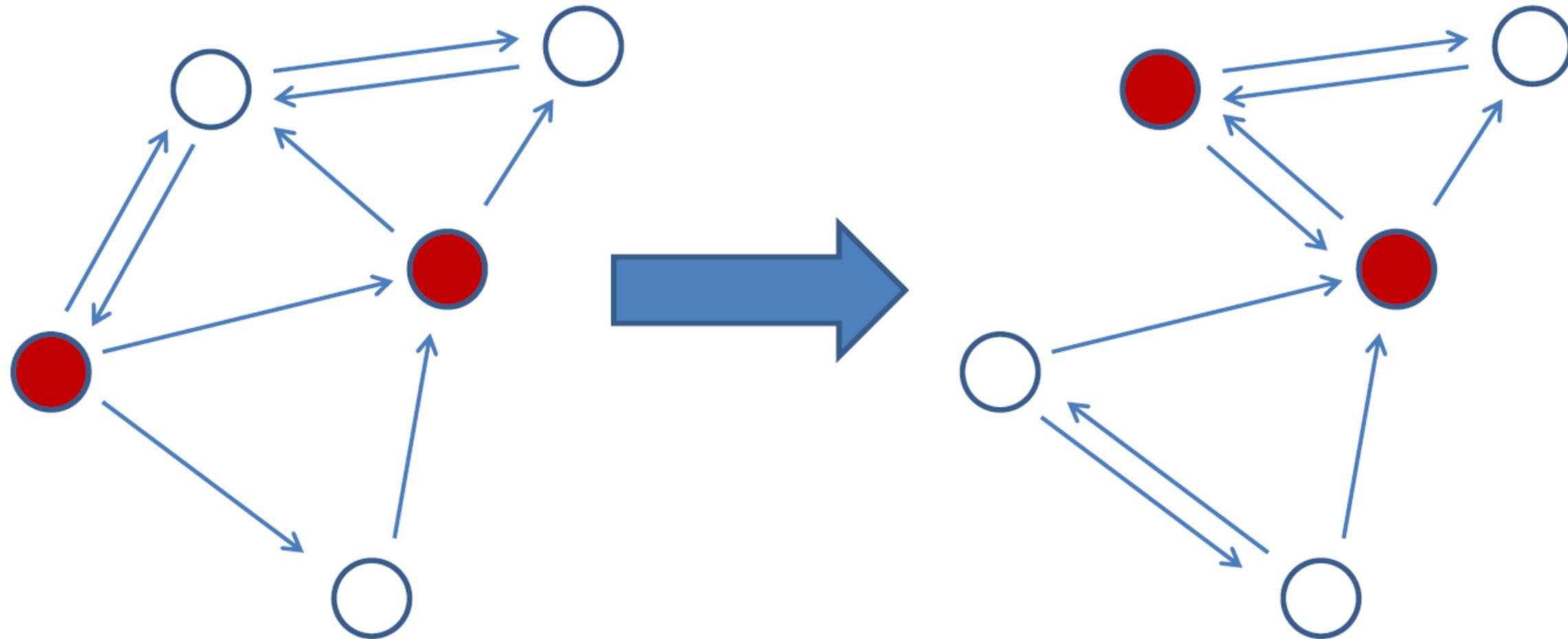
- **Common external causation**: Zak and I are on the Stari Most on 9 November 1993 and jumping is safer than staying on a bridge that is being destroyed by Croat forces

## Influence

- **Biological contagion**: Zak infected me with a virus that makes people jump off bridges

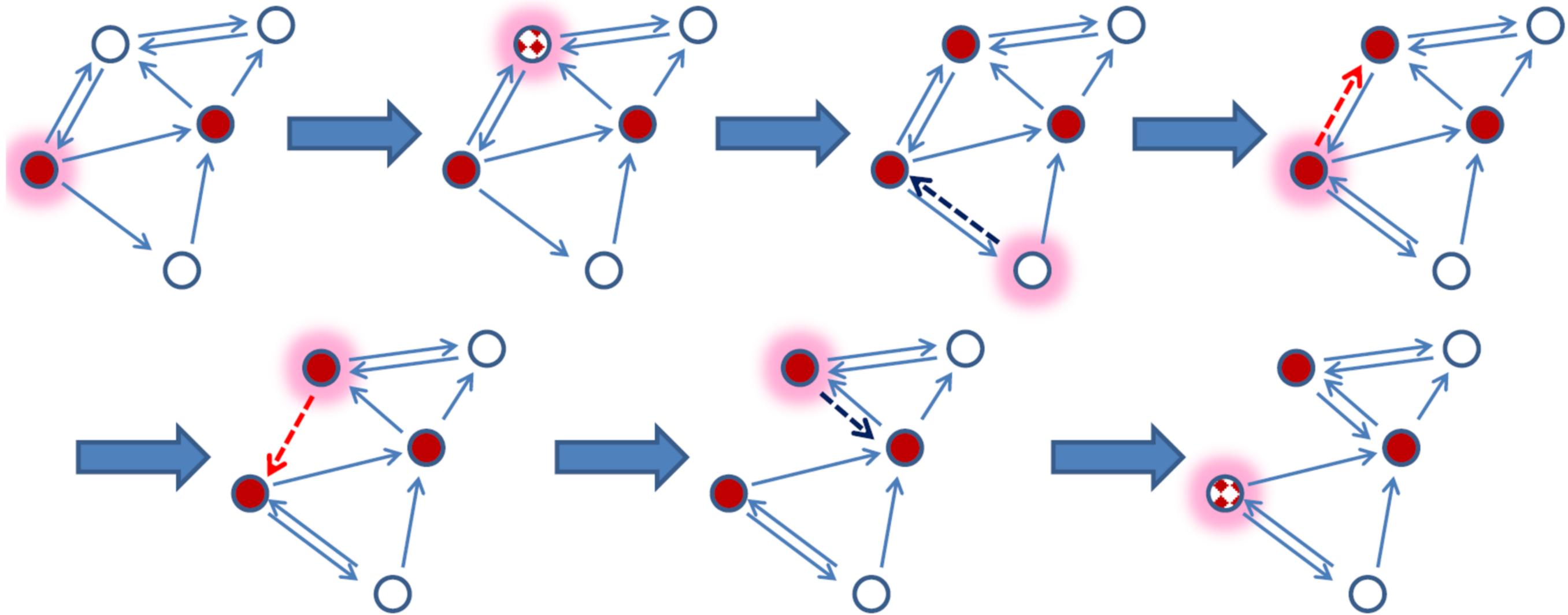
- **Social influence**: Zak inspired me

# Networks and behaviour may change simultaneously



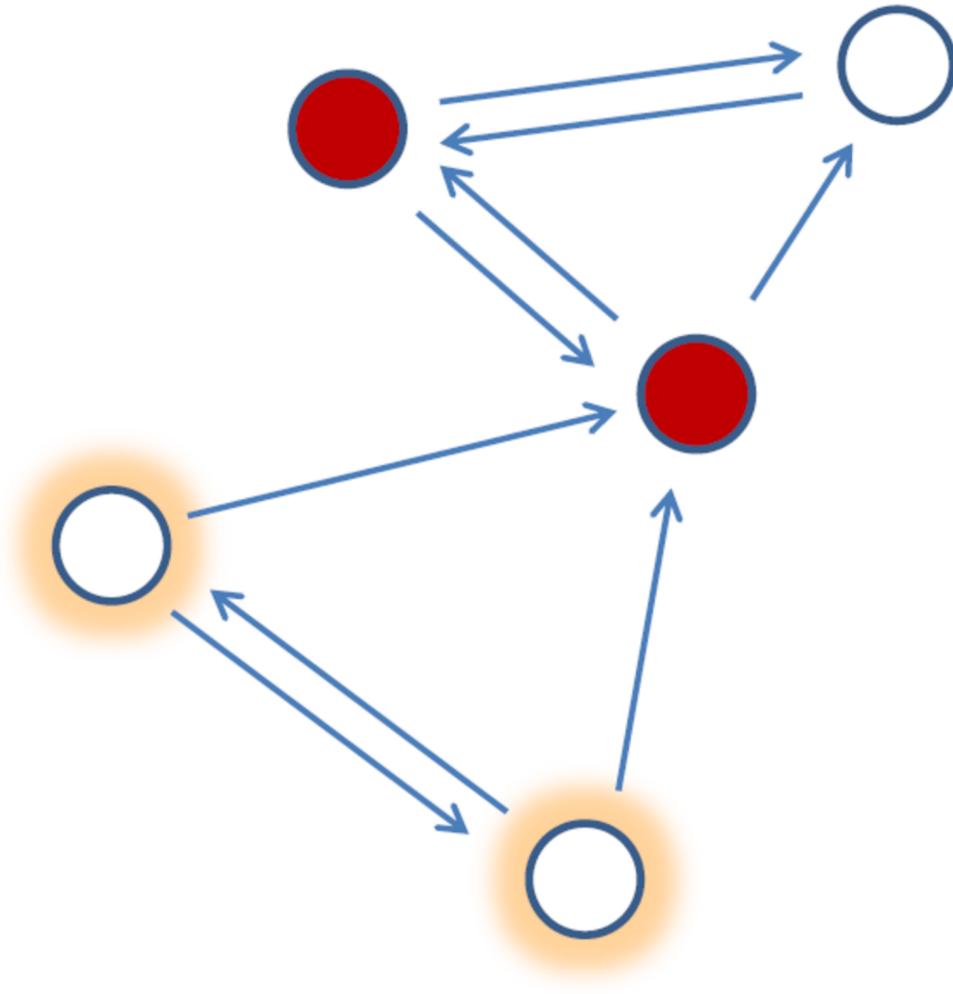
- Still two discrete observations
- Still assume continuous process of change, but now interpolates network-tie changes with behavioural changes

# SAOM allows discrete changes on both levels



- Changes are actor-oriented: individuals decide to change their outgoing ties *and* their behaviour
- Two Poisson processes determine time intervals between subsequent changes in each dependent variable

# Process Markovian (and thus myopic)



- Both highlighted individuals have the same probability to change their behaviour.
- If social influence is present, they might have an increased likelihood to become red.

# Behaviour change is discrete

- Once individuals reconsider their behaviour, they can increase, decrease, or maintain it
- Actual choice modelled with a multinomial probability (up, down, stay)
- This means successive opportunities are required for large-scale behavioural changes
- Model is very similar to network change model



# Individuals evaluate their behaviour

- The objective function:

$$f^{\text{beh}}(i, \mathbf{x}, \mathbf{z}, \boldsymbol{\beta}) = \sum_k \beta_k s_{ki}^{\text{beh}}(\mathbf{x}, \mathbf{z})$$

- The focal actor is  $i$
- $\mathbf{x}$  represents the current network (potentially also other networks and covariates)
- Vector  $\boldsymbol{\beta}$  weights general preferences (social forces)...
- ...that are operationalised with effect statistics  $s_{ki}$
- The objective function can take any real value

# Four example structural effects

Linear tendency:

$$z_i$$

Quadratic tendency:

$$z_i^2$$

Dependence on other covariates:

$$z_i v_i$$

Popularity-related effect:

$$z_i \sum_j x_{ji} = z_i x_{+i}$$

Av. similarity with friends:

$$x_{i+} \sum_j x_{ij} (\text{sim}_{ij}^z - \hat{\text{sim}}^z)$$

# Individuals choose their behaviour level

- Probability of maintain (as compared to increasing or reducing the behaviour level by one) is given by the multinomial probability:

$$P(z_i \rightarrow z^{i\pm}; \mathbf{x}, z, \beta) = \frac{\exp(f^{\text{beh}}(i, \mathbf{x}, z, \beta))}{\sum_{\phi \in \{+, \pm, -\}} \exp(f^{\text{beh}}(i, \mathbf{x}, z^{i\phi}, \beta))}$$

- $z^{i-}/z^{i+}$  equal  $z$  except for actor  $i$  having decreased/increased her behaviour by one;  $z^{i\pm} = z$
- If  $z_i$  has its minimum/maximum value, a further decrease/increase is not possible and the corresponding term in the probability is zero

# SAOMmary

- Network (and behaviour) change is observed across repeated measures
- The discrete change is decomposed into continuous-time ministeps and modelled from an actor-oriented perspective
  - The frequency of these ministeps and which actors are offered an opportunity to change their ties/behaviour is modelled by the rate function
  - What happens during these ministeps/opportunities is modelled by an evaluation function, and the effects included here tend to be most related to research questions
- The Method of Moments estimation procedure seeks to find stable parameter values that simulate networks that match the target statistics of the effects included and are stable (convergence) and also replicate salient macro-structural features (goodness-of-fit)

# The Bestiary of Statistical Network Models



**Theory**

**Data**

---

**Cross-Sectional  
/Panel Data**

(T)ERGMs

SAOMs

---

**Time-Stamped  
Data**

REMs

DyNAMs

## Tie-Oriented

## Actor-Oriented

(T)ERGMs

SAOMs

REMs

DyNAMs

**Tie-Oriented**

**Actor-Oriented**

**Cross-Sectional  
/Panel Data**

(T)ERGMs

SAOMs

**Time-Stamped  
Data**

REMs

DyNAMs

# Advantages



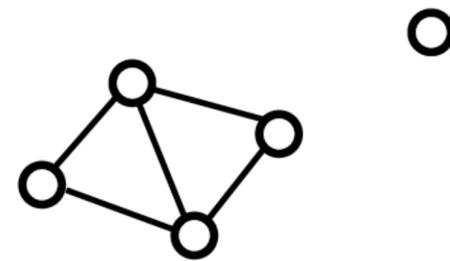
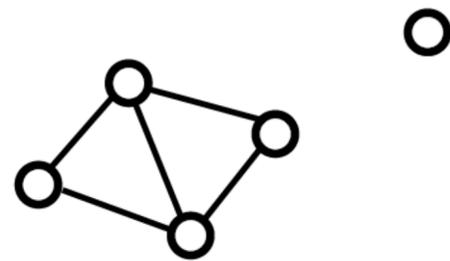
- More **Precision**
  - on sequential inference than panel-based (T)ERGMs or SAOMs where the data is time-stamped
  - on choice inference than tie-based models like REMs because actors' choices separated from opportunity into two functions



# Modelling from actor- or tie-oriented perspective

Actor-based model

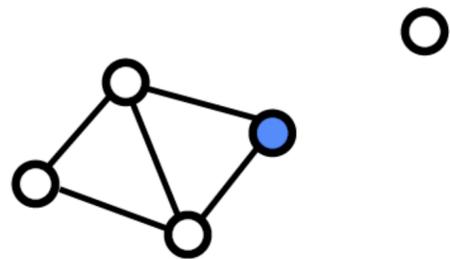
Tie-based model



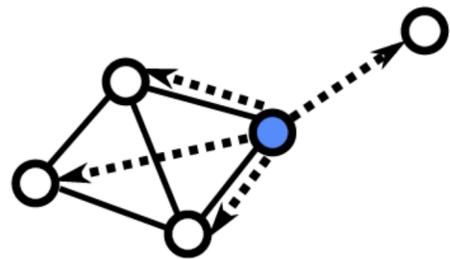
**DyNAM**

“Increase in waiting times”

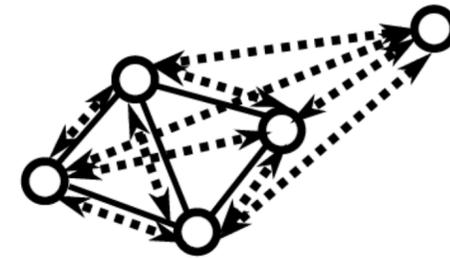
“Increase in relative probability”



1. Competing sender rates



2. Sender chooses receiver



1. Competing tie rates

**REM**  
(Butts, 2008)

Both “increase in waiting times” and “relative probability”

# REM/DyNAM Comparison

- Comparing the rates

$$\lambda_{ij}^{\text{Actor}}(x, \theta, \beta, s, t, A) = \overbrace{\exp(\theta^T s(x, i))}^{\text{step 1: actor rate}} \overbrace{\frac{\exp(\beta^T t(x, i, j))}{\sum_{k \in A} \exp(\beta^T t(x, i, k))}}^{\text{step 2: receiver choice}}$$

$$\lambda_{ij}^{\text{Tie}}(x, \gamma, u, A) = \exp(\gamma^T u(x, i, j))$$

- Conditional probability given actor  $i$  active next

$$P_{i \rightarrow j}^{\text{Tie}}(x, \gamma, A | i \text{ active}) = \frac{\exp(\gamma^T u(x, i, j))}{\sum_{k \in A \setminus \{i\}} \exp(\gamma^T u(x, i, k))}$$

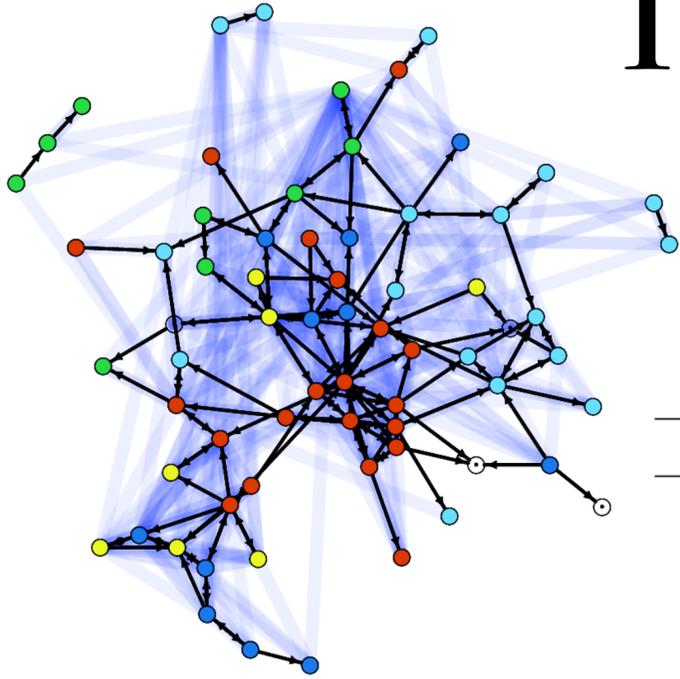
$$P_{i \rightarrow j}^{\text{Actor}}(x, \beta, A | i \text{ active}) = \frac{\exp(\beta^T t(x, i, j))}{\sum_{k \in A \setminus \{i\}} \exp(\beta^T t(x, i, k))}$$

- Probability actor  $i$  active next

$$P_i^{\text{Actor}}(x, \theta, s, A) = \frac{\exp(\theta^T s(x, i))}{\sum_{k \in A} \exp(\theta^T s(x, k))}$$

$$P_i^{\text{Tie}}(x, \gamma, u, A) = \frac{\sum_{j \in A} \exp(\gamma^T u(x, i, j))}{\sum_{k, l \in A} \exp(\gamma^T u(x, k, l))}$$

# The interpretation of parameters differs between models



| #  | Effects                         | DyNAM sender rate |           |     | DyNAM receiver choice |           |     | REM tie rate   |           |     |
|----|---------------------------------|-------------------|-----------|-----|-----------------------|-----------|-----|----------------|-----------|-----|
|    |                                 | $\hat{\theta}$    | s.e.      |     | $\hat{\beta}$         | s.e.      |     | $\hat{\gamma}$ | s.e.      |     |
| 1  | intercept                       | -13.74            | 0.05      | *** |                       |           |     | -14.78         | 0.14      | *** |
| 2  | egoX recentCallsSent            | 0.59              | 0.04      | *** |                       |           |     | 0.55           | 0.03      | *** |
| 3  | egoX recentCallsReceived        | 0.53              | 0.05      | *** |                       |           |     | -0.27          | 0.05      | *** |
| 4  | outdegreeX friendship           | 0.03              | 0.00      | *** |                       |           |     | -0.04          | 0.01      | *** |
| 5  | outdegreeX callNetwork          | 0.26              | 0.01      | *** |                       |           |     | -0.03          | 0.02      |     |
| 6  | outdegree callNetwork           |                   |           |     | -4.09                 | 0.16      | *** | -4.97          | 0.13      | *** |
| 7  | outdegree callNetworkPastHour   |                   |           |     | -2.06                 | 0.19      | *** | 0.79           | 0.11      | *** |
| 8  | reciprocity callNetwork         |                   |           |     | 0.24                  | 0.14      |     | 0.38           | 0.11      | *** |
| 9  | reciprocity callNetworkPastHour |                   |           |     | 4.01                  | 0.32      | *** | 4.67           | 0.13      | *** |
| 10 | inPop callNetwork               |                   |           |     | -0.10                 | 0.05      | *   | 0.01           | 0.03      |     |
| 11 | inPop friendship                |                   |           |     | -0.17                 | 0.02      | *** | -0.06          | 0.01      | *** |
| 12 | transitivity callNetwork        |                   |           |     | 0.30                  | 0.12      | *   | -0.16          | 0.07      | *   |
| 13 | transitivity friendship         |                   |           |     | 0.02                  | 0.04      |     | 0.14           | 0.02      | *** |
| 14 | sameX floor                     |                   |           |     | -0.24                 | 0.12      | *   | -0.98          | 0.08      | *** |
| 15 | sameX gradeType                 |                   |           |     | 0.13                  | 0.12      |     | -0.17          | 0.08      | *   |
| 16 | X friendship                    |                   |           |     | 2.07                  | 0.18      | *** | 1.52           | 0.12      | *** |
|    | Log Likelihood (sub model)      |                   | -14165.94 |     |                       | -1319.38  |     |                | -14800.81 |     |
|    | Log Likelihood (interval)       |                   |           |     |                       | -15485.32 |     |                | -14800.81 |     |
|    | Log Likelihood (sequence)       |                   |           |     |                       | -4987.42  |     |                | -4513.31  |     |
|    | CPU time (seconds)              |                   |           |     |                       | 18.11     |     |                | 2111.69   |     |



# Advantages



- More **Precision**

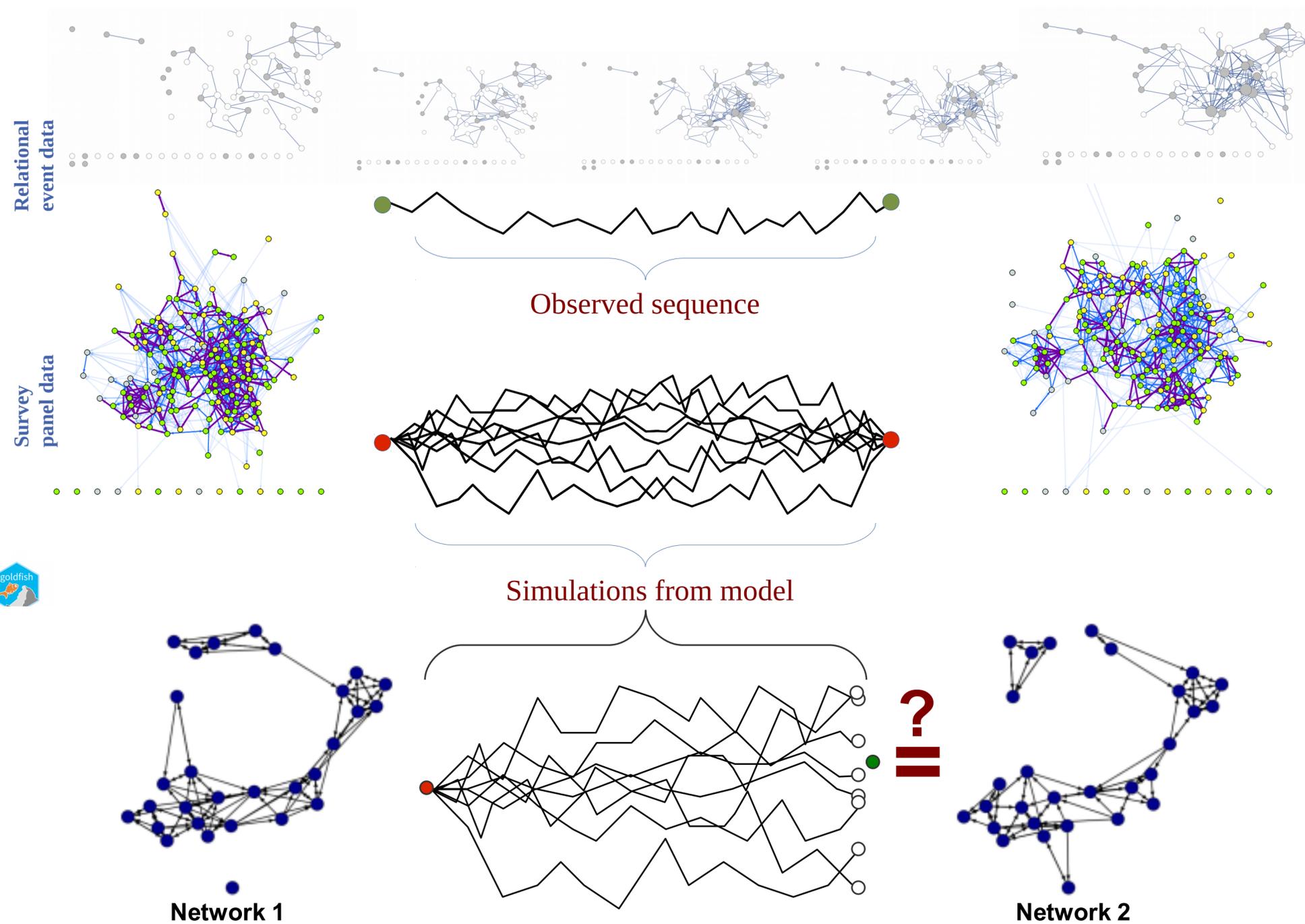
- on sequential inference than panel-based (T)ERGMs or SAOMs where the data is time-stamped
- on choice inference than tie-based models like REMs because actors' choices separated from opportunity into two functions



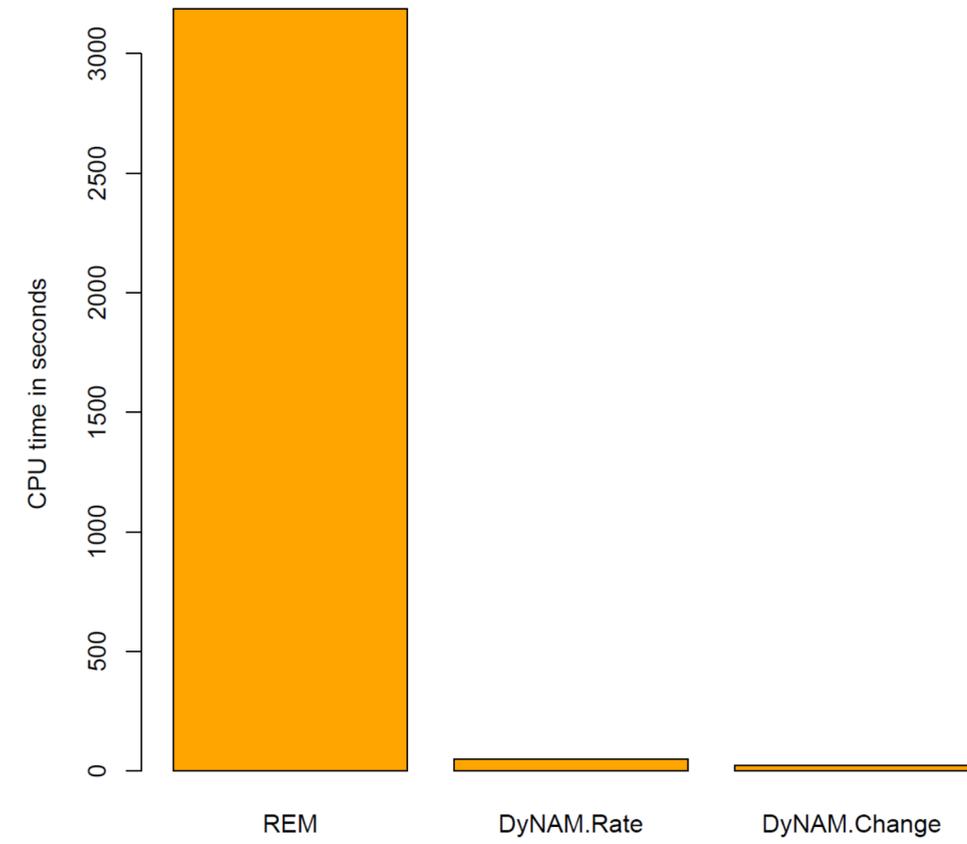
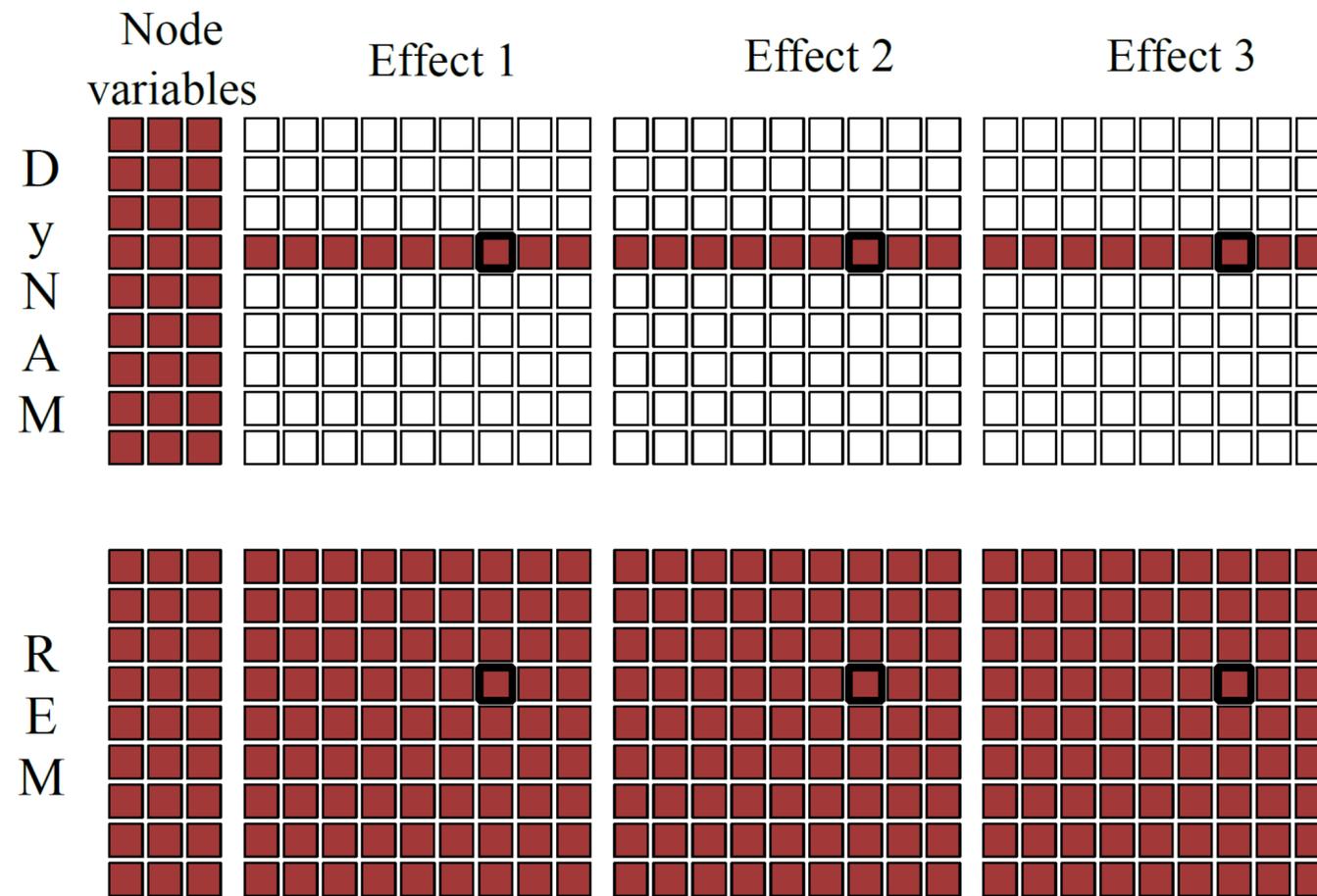
- Better **Performance**

- than (T)ERGMs or SAOMs because does not rely on simulations for estimation
- than REMs because two sub-models means a lower order of computational complexity

# SAOMs and DyNAMs



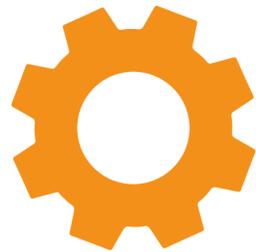
# Models use a different amount of information



# Advantages



- More **Precision**
  - on sequential inference than panel-based (T)ERGMs or SAOMs where the data is time-stamped
  - on choice inference than tie-based models like REMs because actors' choices separated from opportunity into two functions

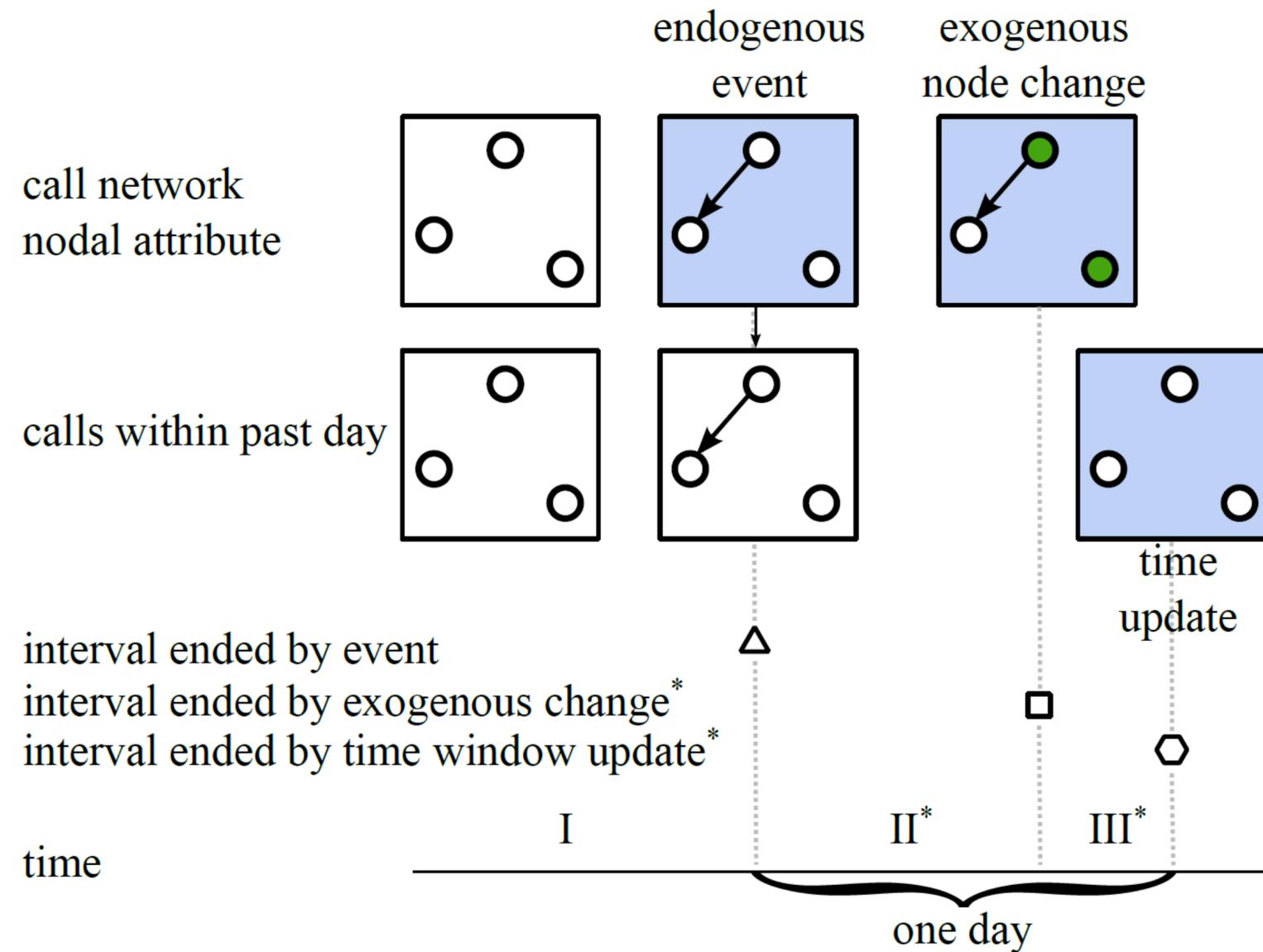


- Better **Performance**
  - than (T)ERGMs or SAOMs because does not rely on simulations for estimation
  - than REMs because two sub-models means a lower order of computational complexity



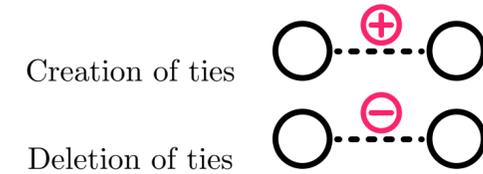
- Additional **Properties**
  - time: windowed and global temporal effects
  - weights: binary/multiple and weighted effects

# Time windows

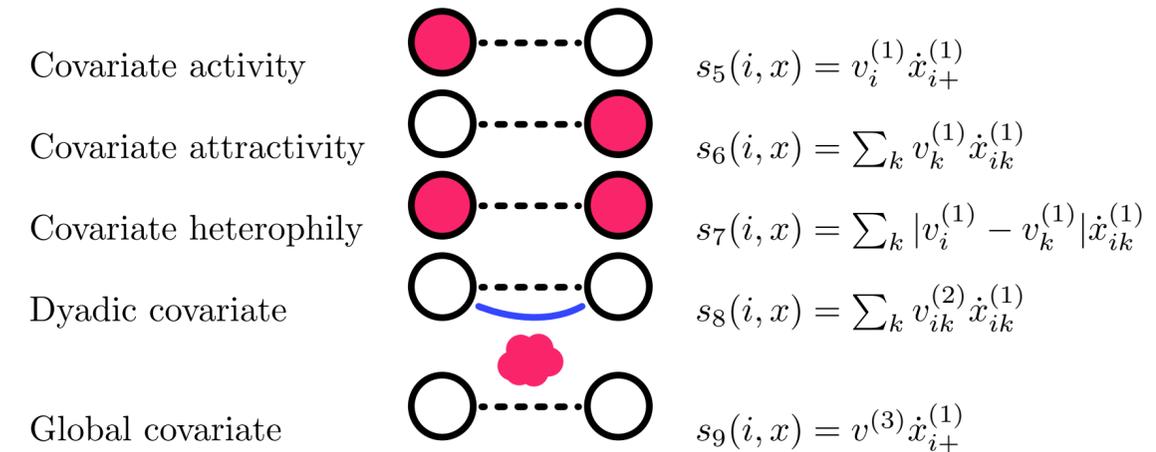
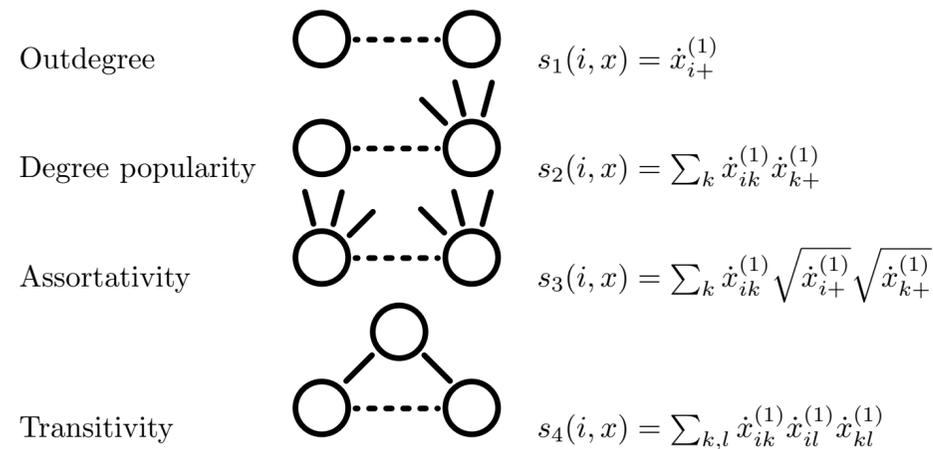


# Effects Smörgåsbord

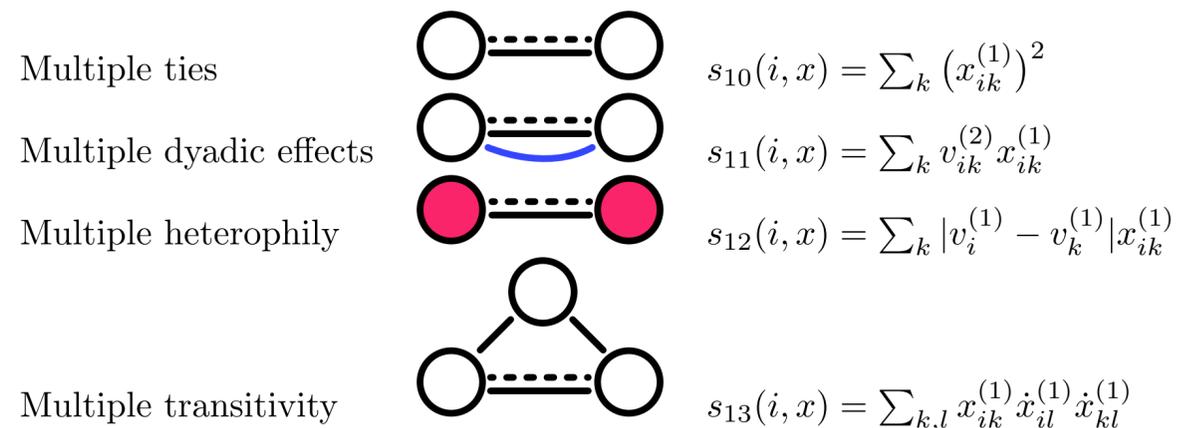
## Signed



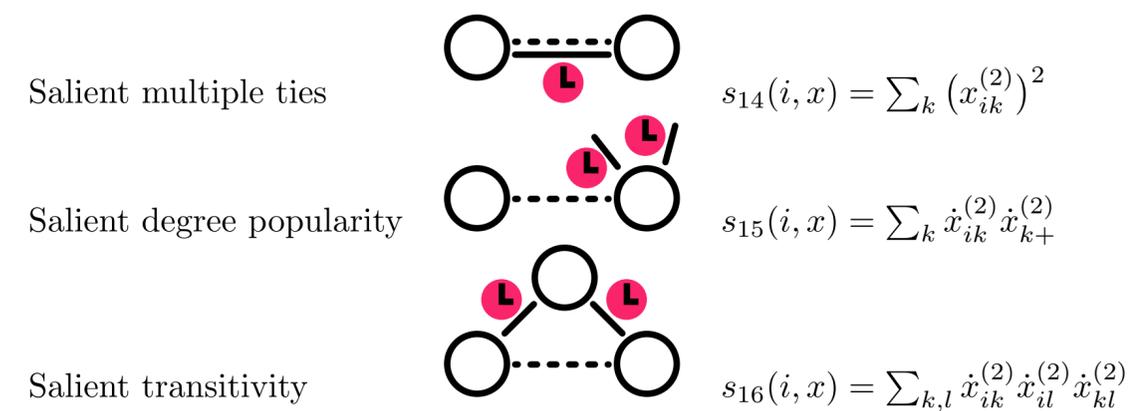
## Generic



## Multiple



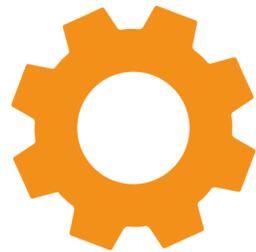
## Windowed



# Advantages



- More **Precision**
  - on sequential inference than panel-based (T)ERGMs or SAOMs where the data is time-stamped
  - on choice inference than tie-based models like REMs because actors' choices separated from opportunity into two functions



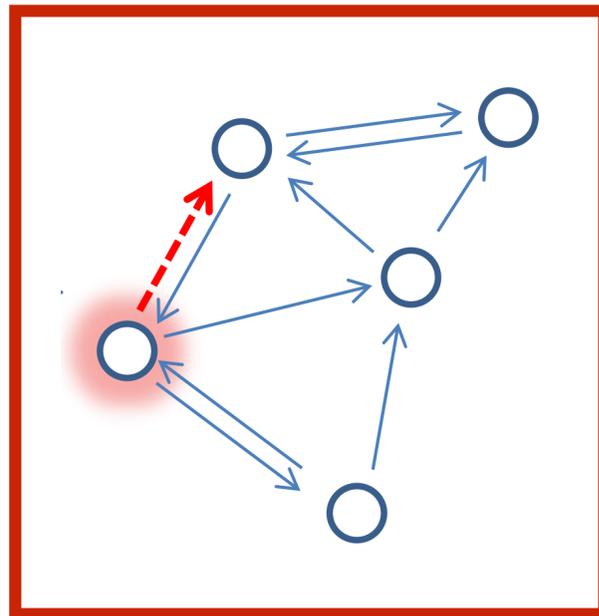
- Better **Performance**
  - than (T)ERGMs or SAOMs because does not rely on simulations for estimation
  - than REMs because two sub-models means a lower order of computational complexity



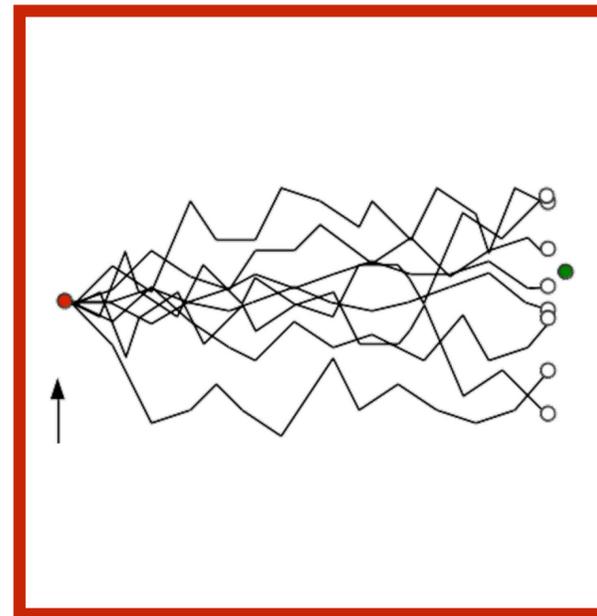
- Additional **Properties**
  - time: windowed and global temporal effects
  - weights: binary/multiple and weighted effects

# SAOM

Model



Estimation



Influence

