

**GENEVA  
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INSTITUT DE HAUTES  
ÉTUDES INTERNATIONALES  
ET DU DÉVELOPPEMENT  
GRADUATE INSTITUTE  
OF INTERNATIONAL AND  
DEVELOPMENT STUDIES

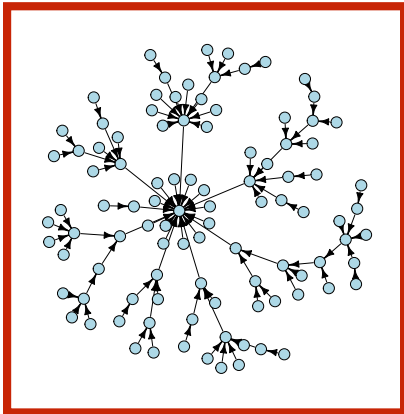
# Topology

## Introduction to Social Networks

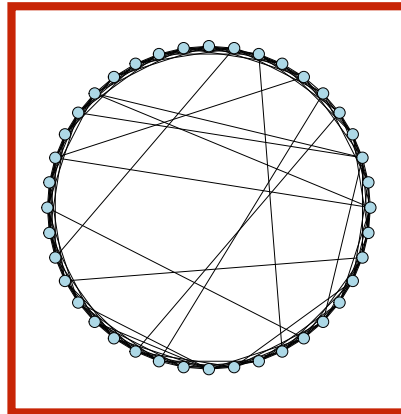
James Hollway

# Topology

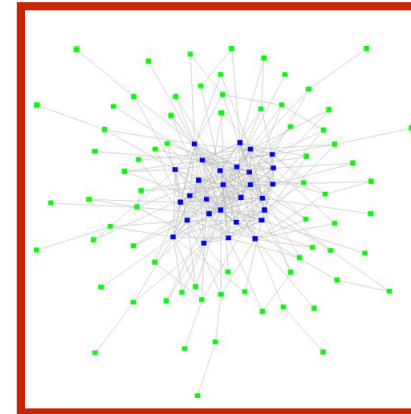
Scale-Free

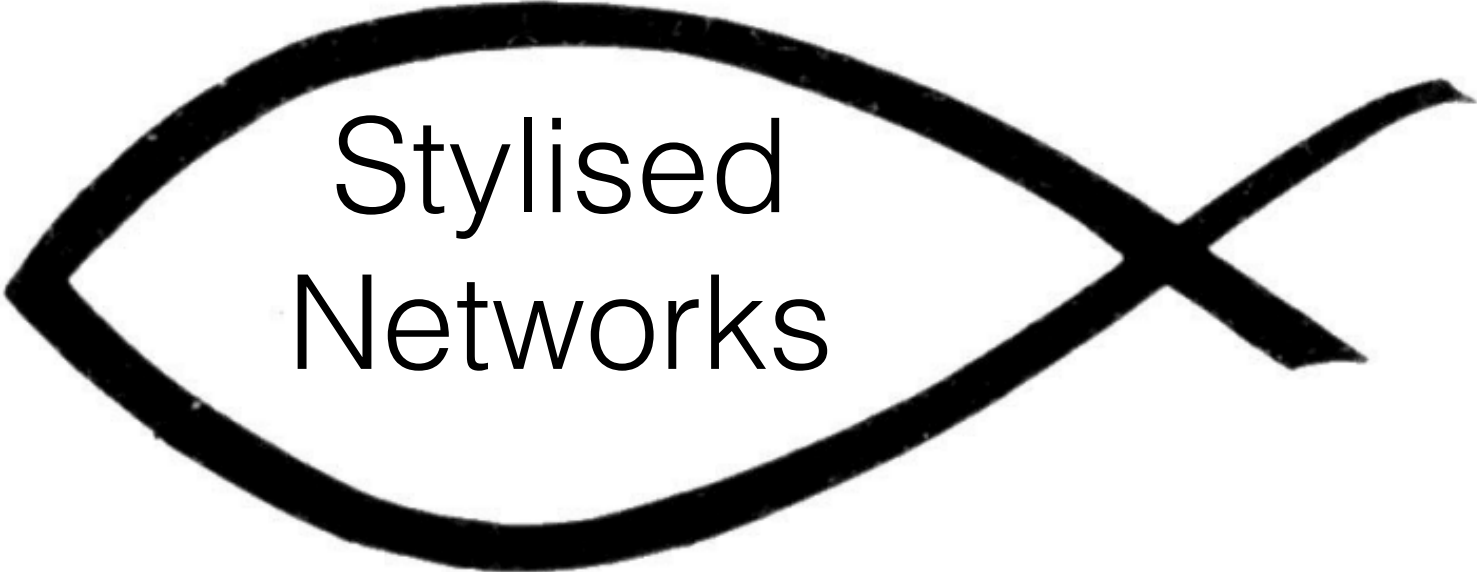


Small-World



Core-Periphery



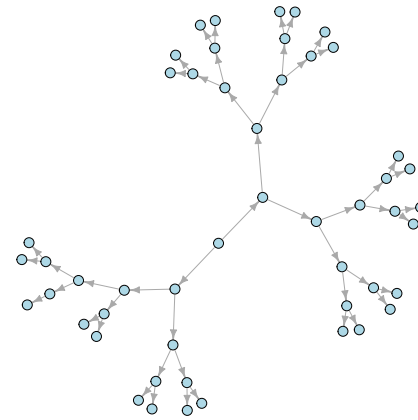


Stylised  
Networks

# What are stylised network models?

- **Ideal type** networks
  - Constructed according to 1-2 simple rules
  - Exaggerate structural features commonly found in networks
    - Centrality
    - Cohesion
    - Randomness

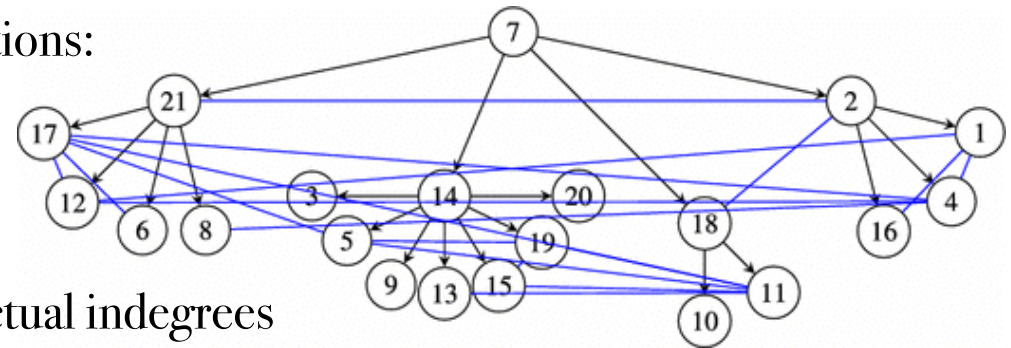
# 1. Regular Tree



- Rationale:
  - Some networks tend to be **centralised**, i.e. some nodes have better reach than others and/or network is asymmetric
  - Often the case in asymmetric, functional, or hierarchically organised settings
- Generated by creating a network of *branching* nodes with parameters:
  - Number of branches per node (here 2)
  - Network distance (generations) or dimensions (here  $n=50$ )
- Uses: some use in organizational and biological networks

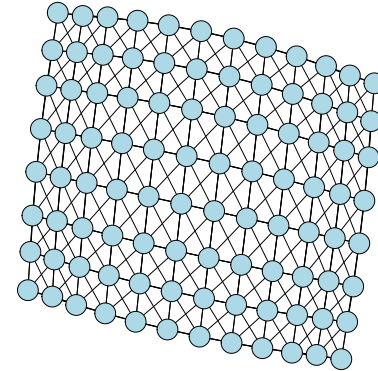
# Graph theoretic dimensions (GTD) of hierarchy

- Hierarchy common in networks/ life (Simon 1981), but while intuitive, difficult to be precise
- Krackhart (1994) provided an elegant definition of ideal typical hierarchy as an out-tree graph, where all nodes connected and all but one (the 'boss') has an in-degree of one.
- Four individually necessary and jointly sufficient conditions:
  - **Connectedness**: proportion of dyads reachable
  - **Hierarchy**: inverse of reciprocity
  - **Efficiency**: sum of minimum indegrees over sum of actual indegrees
  - (Least) **upper bound**: a node that can reach a pair of other nodes (lub is an upper bound that is included on at least one directed path from every other upper bound to each of  $x$  and  $y$ )



Krackhardt 1994, Everett and Krackhardt 2012

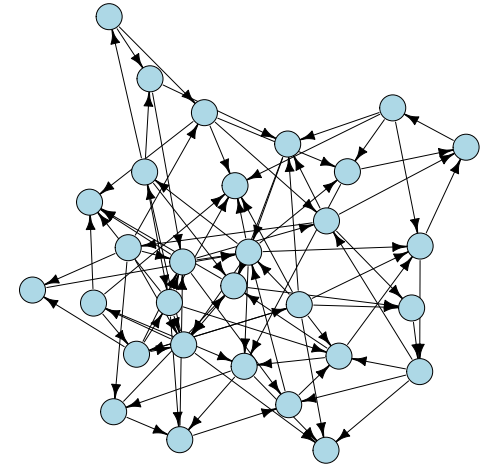
# 2. Regular Lattice



- Rationale:
  - Some networks tend to be **clustered**, i.e. high probability that one's interaction partners also interact
  - Often the case where geographic or social space important
- Generated by arranging nodes on a **lattice** with parameters:
  - Can vary by number of dimensions (here 2)
  - And neighbourhood/interaction distance (here 1+diagonal)
- Uses: commonly used in ABM to show how spatial or network clustering can allow or limit diffusion or make pockets of behaviour stable (more next week)

# 3. Random network

- Also known as a *Bernoulli* network (after Swiss mathematician Jacob Bernoulli, brother to Johann, Euler's advisor) or an *Erdős-Renyi* (1959; see also Rapaport 1953) network
- Rationale:
  - The opposite of a structured network is a **random network**, i.e. where each tie has an equal probability of existing
  - Rarely the case that an empirical network is truly random, but used as below
- Generated by creating edges at random on a network with parameters:
  - Nodes (here 30)
  - Density
- Uses: often used as a simple baseline to ascertain whether a certain substructure observed more often than expected by chance



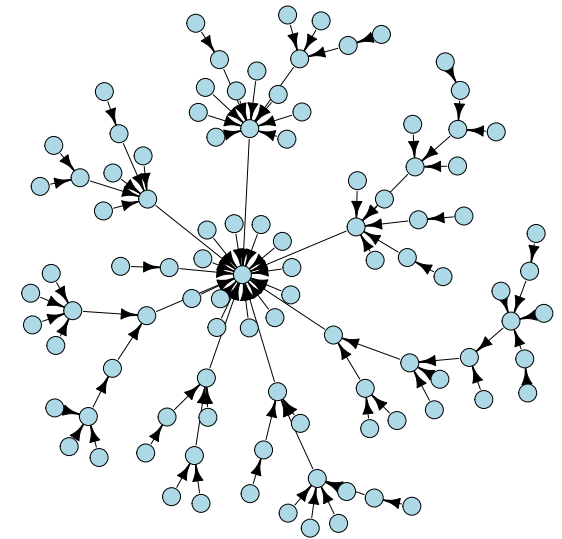
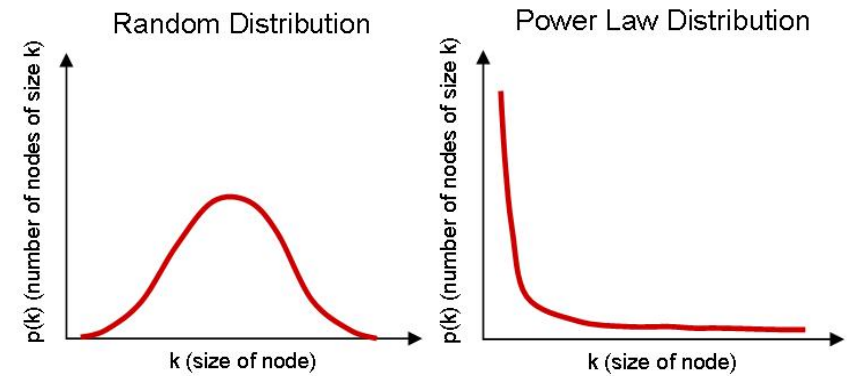


# The Matthew Effect

- In 1968, Robert Merton published an article called “The Matthew Effect in Science”
  - “For unto every one that hath shall be given, and he shall have abundance; but from him that hath not shall be taken even that which he hath” Matthew 25:29
- In this article, he discussed how academic fame leads to more fame – in terms of prizes, citations, and attribution of merit (**cumulative advantage** or **preferential attachment**)
- In citation networks this mechanism in its purest form can lead to so-called scale-free networks

# 4. Scale Free

- Rationale:
  - Some networks have a strong *degree dispersion*
  - Often the case where *positive feedback mechanisms* prevalent (e.g. internet, twitter, academic papers)
- Generated by iterative creation of a network where each new node ties to existing nodes with probability proportional to their degree
  - Degree distribution of this generative process follows a power law
- Uses: some scholars claim power laws are common feature of many networks



De Solla Price 1965; Barabási and Albert 1999

# The Milgram Experiment

- In the 60s, Stanley Milgram did an experiment where he invited (through advertisement) people to send a letter to a person unknown to them through intermediaries
- He found that everybody was connected to everyone else through 6 steps
  - He thought this was evidence we lived in a “small world”
- What was wrong with his experiment?

# What is wrong with Milgram's (small) world?

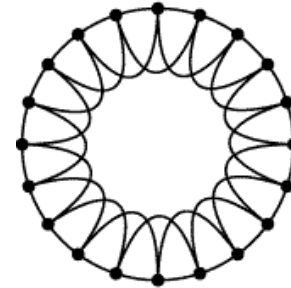
- **Selection bias**: “starters” were recruited through an advertisement searching for well-connected people
- **Non-response bias**: if one assumes an attrition rate, longer chains will be underrepresented
- **Greedy algorithm**: people can only make local decisions and cannot omnisciently recognise the shortest global path
- But, Watts and Strogatz (1998) showed this using computer simulations and rise of relational rewiring has brought “6 degrees of separation” into popular culture

See also Connor and Simberloff 1979

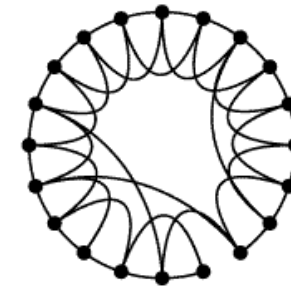
# 5. Small World

- Rationale
  - Some networks tend to be clustered by interconnected by just a few spanning ties
  - Often the case across a surprising range of settings
- Generated by creating a (ring) lattice and then rewiring a few ties at random with parameters:
  - Lattice dimensions and distance
  - Probability of a tie being rewired
- Uses: commonly used to show “it’s a small world after all” and to model diffusion

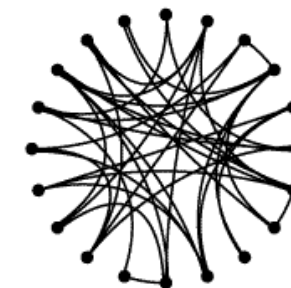
Regular



Small-world

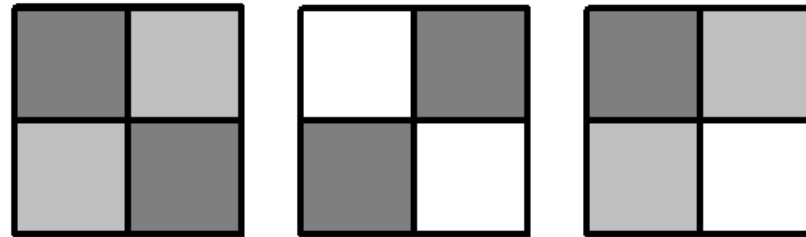


Random

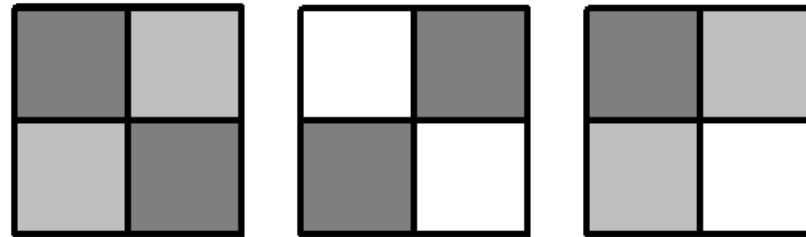




# Choose the odd one out



# Pick the odd one out

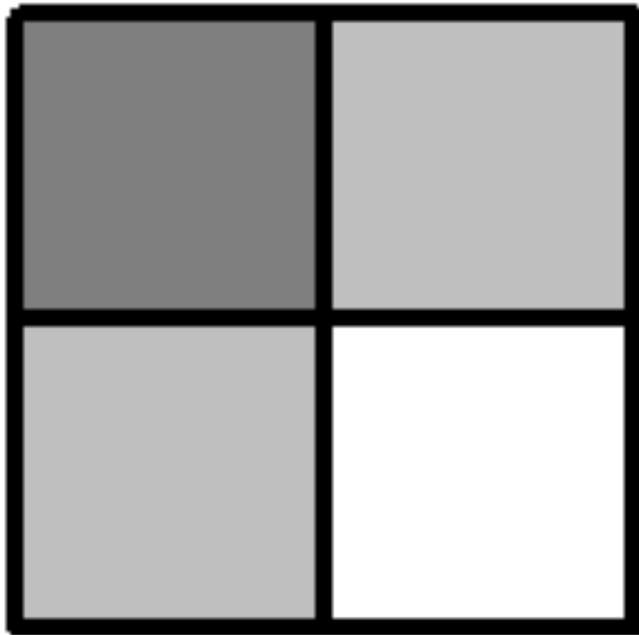


- The idea that at a *meso*-level there are more centralised networks with a *group* of central/core nodes and then others around them is a potentially theoretically relevant observation for many networks
  - e.g. world systems theory (Wallerstein 1974; 2004; Snyder and Kick 1979)
- Various methods for extracting core-periphery structures: statistical inference, spectral decomposition, diffusion mapping, motif counting, geodesic tracing, model averaging

Van Lidth de Jeude et al (2018); see also Prell et al. (2023)



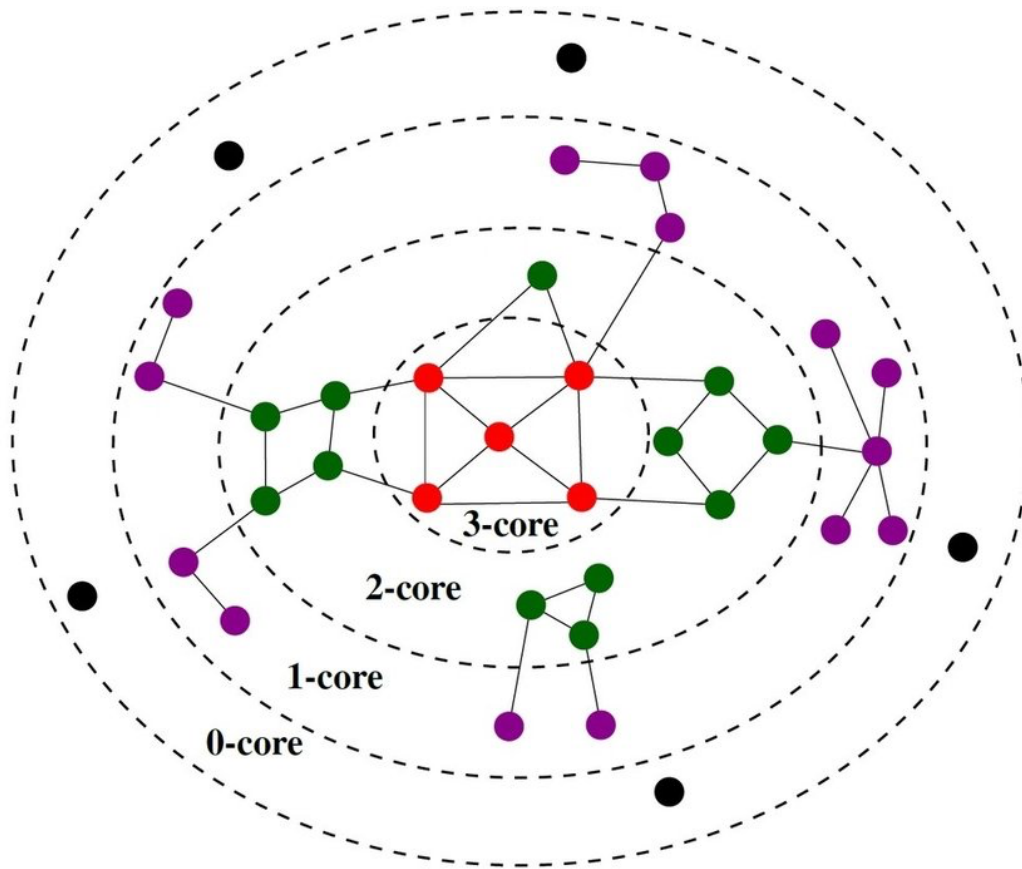
# Classic ‘hub-and-spoke’ two-block model



- A fundamental network pattern: dense “core” of tightly connected nodes, connected less densely to peripheral nodes, which are themselves sparsely connected
- Two groups: “core nodes are adjacent to other core nodes, core nodes are adjacent to some periphery nodes, and periphery nodes do not connect with other periphery nodes”
- A central hub and a periphery that radiates out from that hub, gets at *core-as-density*

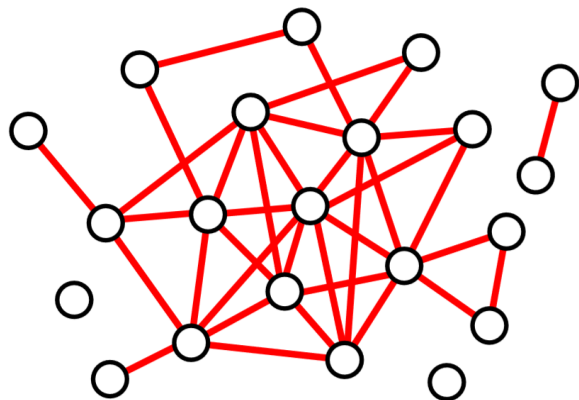
Borgatti and Everett 1999; see also Zhang et al 2015, Kojaku and Masuda 2017, Rombach, Porter et al. 2014

# Alternative: layered coreness

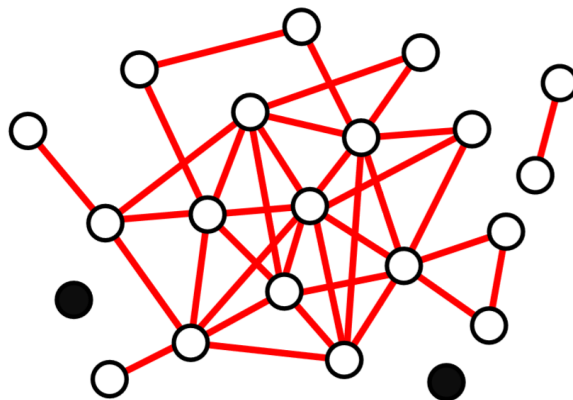


- Alternatively, some instead look at  $k$ -cores
- The largest subset of nodes in the network such that every node has at least  $k$  connections to other nodes in the  $k$ -core but not the  $(k+1)$ -core
- Periphery described as “shells”, “onion layers”, “leaves”, and core “epicenter”, “corona”, or “nucleus”
- Advantages that it is scalable and gets at *core-as-nesting*

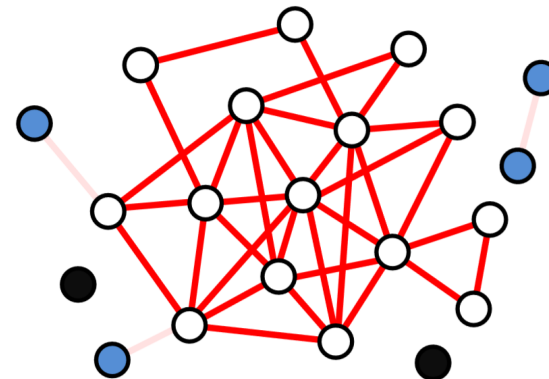
Original network



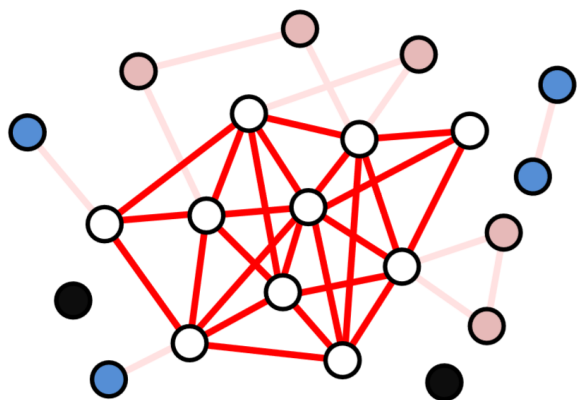
$k = 0$



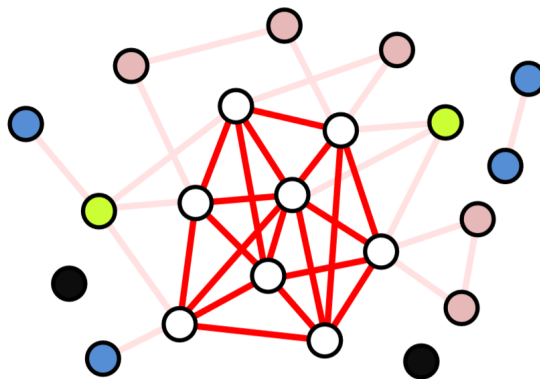
$k = 1$



$k = 2$

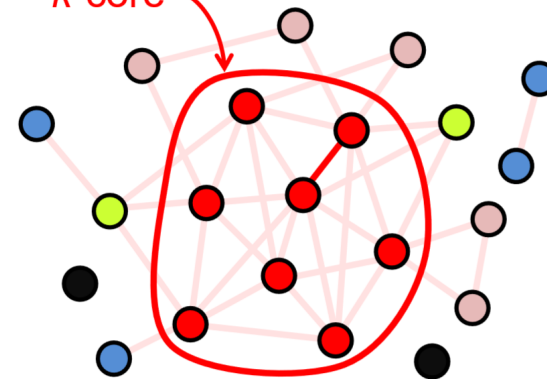


$k = 3$



$k = 4$

*k*-core



Coreness

● : 0

● : 1

● : 2

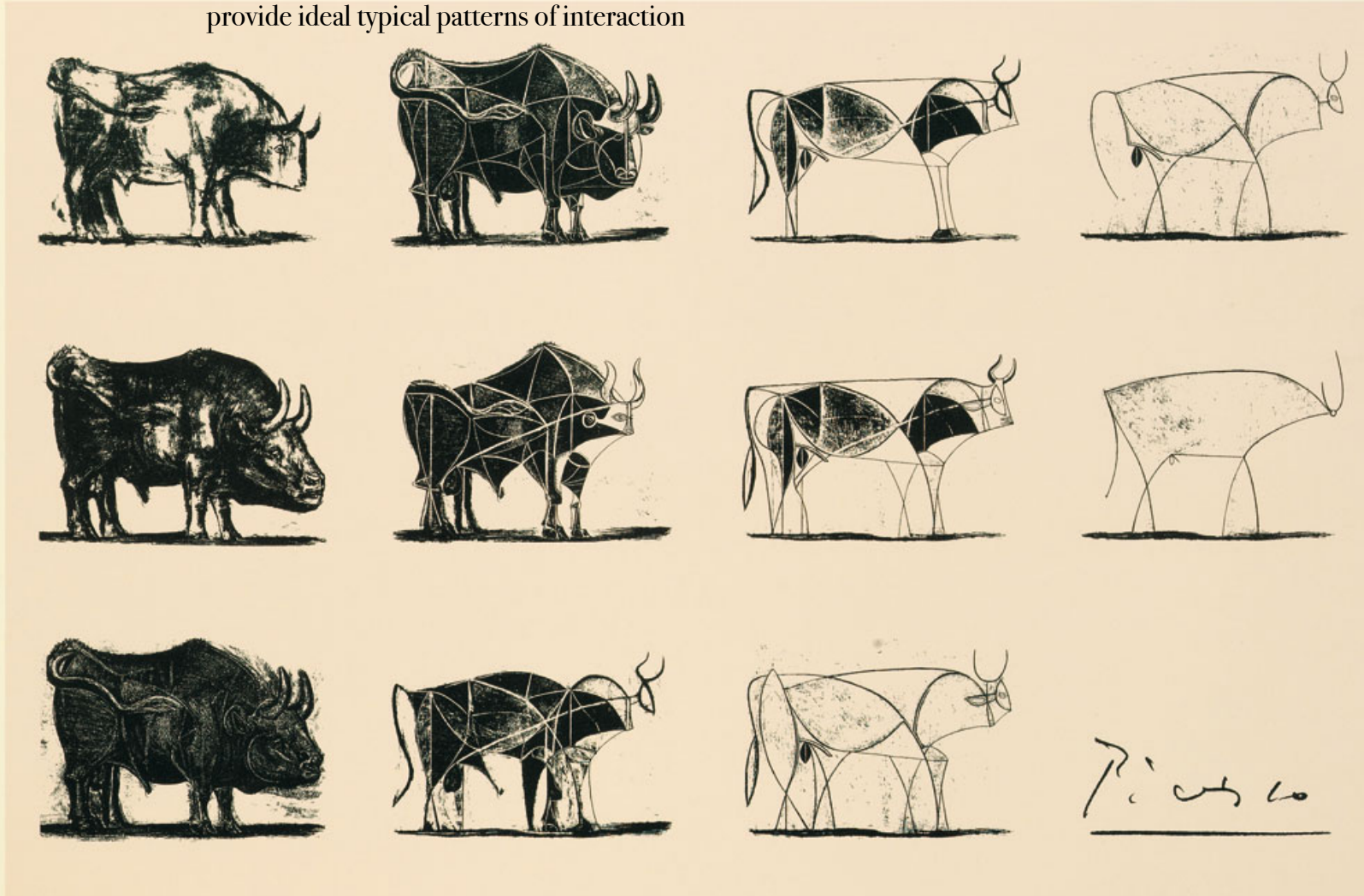
● : 3

● : 4



# Stylised network models

provide ideal typical patterns of interaction



# Stylised network models

provide **ideal typical** patterns of interaction

- Can illuminate how micro mechanisms create macro structures
- Good for theory-building and understanding that structure matters

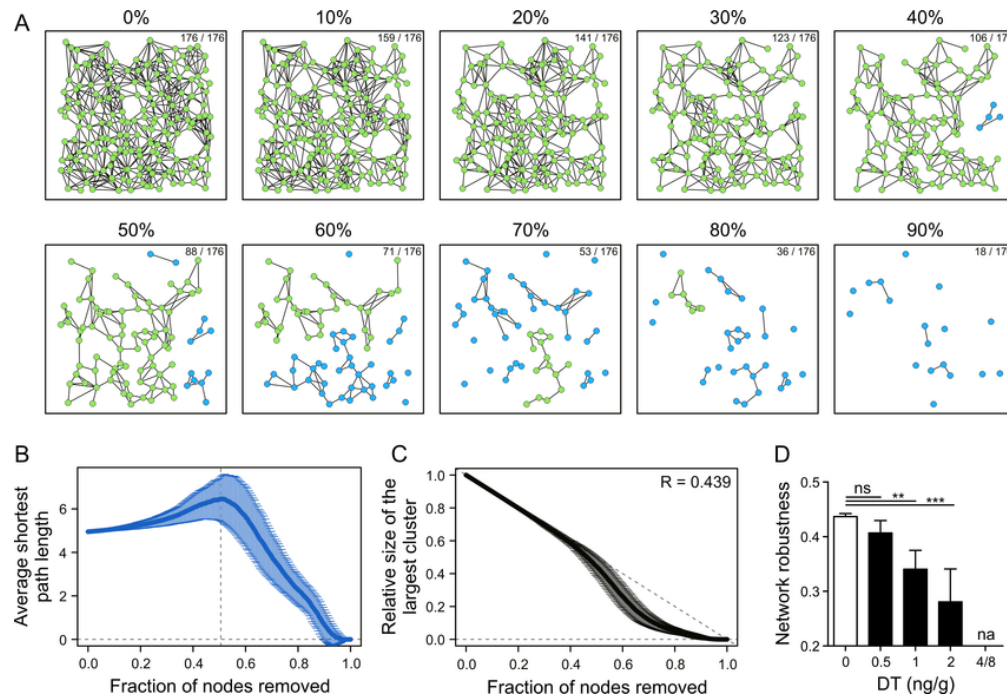
# Stylised network models

provide **ideal typical** patterns of interaction

- Can illuminate how micro mechanisms create macro structures
- Good for theory-building and understanding that structure matters
- These mechanisms at best capture only part of the story
- Little use in comparing empirical and stylised networks and claim that a “fit” means the network is explained



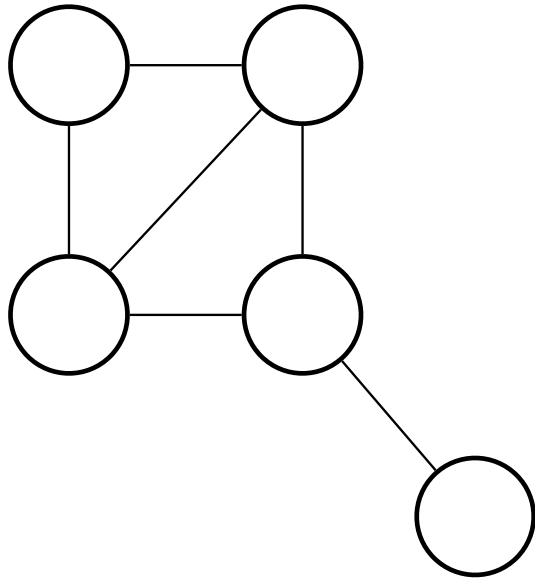
# Network Robustness



**Network percolation theory** is a literature that tries to identify how many random or specific nodes can be removed before a network breaks into multiple components



# Cohesion and Adhesion

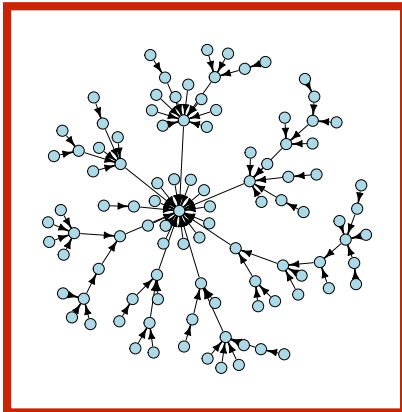


- **Cohesion** is a count of the number of *nodes* that would need to be dropped for the number of components to increase
- **Cutpoints** are the nodes that, if dropped, would result in the number of components to increase
- **Adhesion** is a count of the number of *ties* that would need to be dropped for the number of components to increase
- **Bridges** are the ties that, if dropped, would result in the number of components to increase

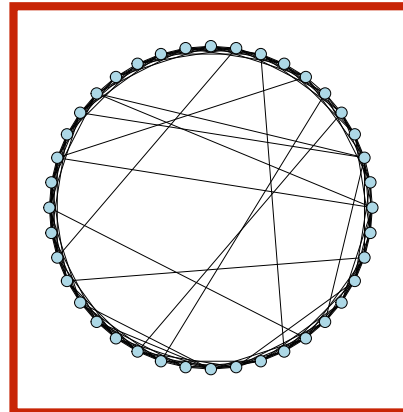


# Topology

Scale-Free



Small-World



Core-Periphery

